TECHNICAL REPORT M-68-1

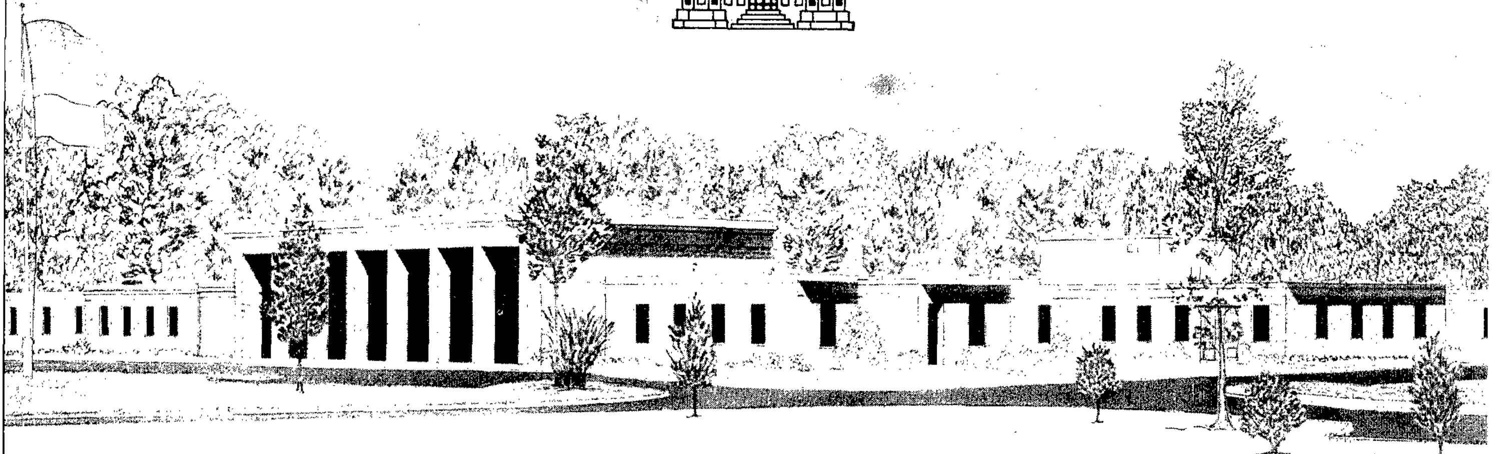
DYNAMICS OF WHEELED VEHICLES

Report 3

A STATISTICAL ANALYSIS OF TERRAIN-VEHICLE-SPEED SYSTEMS

by

N. R. Murphy, Jr.



April 1971

Sponsored by U. S. Army Materiel Command

Conducted by U. S. Army Engineer Waterways Experiment Station, Vicksburg, Mississippi

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FOREWORD

This report was prepared by Mr. Newell R. Murphy, Jr., of the Research Projects Group, Mobility Research Branch, Mobility and Environmental Division, U. S. Army Engineer Waterways Experiment Station. The report is essentially a thesis submitted by Mr. Murphy in partial fulfillment of the requirements for the degree of Master of Science in Engineering to the Faculty of Mississippi State University, and is a study concerned with vehicle dynamics. The study described herein was conducted under DA Project 1T062103A046, "Trafficability and Mobility Research," Task 03, "Mobility Fundamentals and Model Studies," under the sponsorship and guidance of the Research, Development and Engineering Directorate, U. S. Army Materiel Command. The study was accomplished under the general direction of Messrs. S. J. Knight and W. G. Shockley.

COL Levi A. Brown, CE, and COL Ernest D. Peixotto, CE, were Directors of the Waterways Experiment Station during the period of preparation and publication of this report. Mr. F. R. Brown was Technical Director.

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The author is grateful to the U. S. Army Corps of Engineers, Office of the Chief of Engineers and to the U. S. Army Materiel Command for permission to use the information presented herein. Credit is also due the author's supervisors of the U. S. Army Engineer Waterways Experiment Station for their staunch support of education and professional excellence among their subordinates. In addition, the author extends his gratitude to his faculty advisor, Dr. Walter Carnes, and the remainder of his graduate committee, particularly to Dr. Allan S. Lessem, On-Site Thesis Advisor, for his counsel and guidance during this study. Special acknowledgment is offered to Mr. R. B. Ahlvin and Mr. S. I. Kang for their assistance in the computer programming required for this study. The assistance of Mrs. Jane N. Brown, as typist, and Mrs. Edna P. Roberts, in helping prepare the figures for this thesis, was invaluable.

Last but not least, the author wishes to express his sincere appreciation to his wife, Althea, and children, Debra, George, and Rogers, for the patience and understanding they exhibited during this study and for whom this work is dedicated.

N. R. M.

Vicksburg, Miss.
December 1970

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CHAPTER I: INTRODUCTION

In recent years the concept of speed made good has come to be recognized as one of the most meaningful measures of the off-road mobility of a vehicle. Speed made good is simply the average speed that results when the straight-line distance between two points is divided by the time required to get from one point to the other, irrespective of the actual path taken by the vehicle. The maximum speed that a vehicle can attain in a given terrain is affected by the number and character of the terrain deterrents it must overcome or, if preferable, avoid. Terrain deterrents include soft soils, slopes, vegetation, obstacles, riverine situations, rough terrain, etc.

Experience has shown that one of the most significant single features influencing a vehicle's speed is that of ride dynamics, that is, the vibratory activity of a vehicle caused by terrain roughness. Because of this, a great deal of interest has been shown in the ride problem. However, to date, there is no satisfactory means of relating a measure of ride to a measure of terrain roughness. There has actually been much work done in the area of random vibration studies, and the literature available is quite abundant. However, for the most part, this work is largely of an academic nature and oriented toward analysis of linear systems using the concepts of transfer functions. The results are very general and nebulous in nature, and an adequate understanding of them by an individual would require a substantial background of education and experience in engineering, physics, mathematics, statistics, and electronics. It is for this reason that widespread practical application of these results has been stymied.

The General Problem

The description and evaluation of vehicle dynamic responses involves essentially a study and interpretation of waveforms. Generally, an arbitrary waveform (the terrain) serves as an input to excite some deterministic system (a vehicle or a model), and the outputs are generated as continuous waveforms representing the resulting motions of the system. Obviously, these continuous traces or waveforms must be analyzed in accordance with some specified standard to provide a basis for comparisons. This means simply that numbers must be determined that adequately describe the pertinent characteristics of each waveform. Generally, these waveforms are not well-defined mathematical functions, such as sinusoids, sawtooth waves, etc., but vary in seemingly erratic fashions. However, regardless of how erratic these waveforms may appear, their peculiar characteristics can, possibly, be described adequately by statistical methods.

Most engineers are familiar enough with the fundamentals of statistics to know that statistical quantities evolve theoretically from idealized mathematical relations, such as infinite integrals, that serve as approximations to the real world. Therefore, when statistics is applied to real-world problems, only estimates of the true values can be obtained, and the deviation of these estimates from the true values depends on how well certain criteria have been met. For example, when a coin is flipped a sufficient number of times, about as many heads as tails result. This relation improves as the number of flips is increased, and theoretically the number of heads and tails is the same after an infinite number of flips.

The nature of the quantities one might choose to describe vehicle-terrain responses and how they may be interpreted to define certain properties that have physical meaning in mobility studies are discussed here without rigorous details of probability and sampling theory.

Averages, measures of mean, and dispersion

An average is a value that is typical or representative of a set of data. Since such typical values tend to lie centrally within a set of data arranged according to magnitude, averages are also called measures of central tendency. Some of the most commonly used averages are the mean, the median, and the mode.

Variation or dispersion is a measure of the degree that numerical data tend to spread about some reference value. Generally, this reference is chosen to be the mean. Several commonly used measures of variation or dispersion are the mean deviation, the standard deviation, the variance (which is simply the square of the standard deviation), the mean square, and the root mean square (RMS). As an example of how such values represent data, examine the four groups of numbers in the following illustration:

(1)		(2)		(3)		(4)	
7.5	56.25	7	49	5	25	2	4
7.5	56.25	8	64	10	100	13	169
7.5	56.25	7	49	6	36	4	16
7.5	56.25	8	64	7	49	8	64
7.5	56.25	7	49	9	81	10	100
7.5	56.25	8	64	8	64	8	64
45.0	337.50	45	339	45	355	45	417
$\frac{45.0}{6} = 7.5$ (arithmetic mean)		$\frac{45}{6} = 7.5$ (arithmetic mean)		$\frac{45}{6} = 7.5$ (arithmetic mean)		$\frac{45}{6} = 7.5$ (arithmetic mean)	
$\frac{337.5}{6} = 56.25$ (mean square)		$\frac{339}{6} = 56.50$ (mean square)		$\frac{355}{6} = 59.16$ (mean square)		$\frac{417}{6} = 69.50$ (mean square)	
RMS = 7.50		RMS = 7.52		RMS = 7.69		RMS = 8.34	

All four groups have the same arithmetic mean, and obviously, the numbers in groups (1) and (2) are more uniform than those corresponding in (3) and (4). It is, therefore, apparent that the arithmetic mean gives only some measure of central tendency and provides no indication of the uniformity of numbers. However, the numbers in the right-hand columns in the above groups reflect the squared values of the left-hand numbers, and the mean of these numbers (the mean square) is not the same in all columns, but increases in magnitude with an increase in nonuniformity. Hence, the mean square value gives some measure of nonuniformities. Thus, the square root of these mean square values (RMS) would yield a similar descriptor of nonuniformity.

Suppose now that the four left-hand columns of numbers have different mean values. To provide a number with a more consistent comparison capability, the deviation about each mean value can be found. This quantity is referred to as the mean deviation. To illustrate this, consider again the four left-hand columns, each with an arithmetic mean of 7.5. The mean deviation for group 1 is

$$\frac{|7.5-7.5|+|7.5-7.5|+|7.5-7.5|+|7.5-7.5|+|7.5-7.5|+|7.5-7.5|}{6} = 0 \quad (1)$$

i.e., there is zero dispersion or variation of the numbers about the arithmetic mean. However, for group 4

$$\frac{|2-7.5|+|13-7.5|+|4-7.5|+|8-7.5|+|10-7.5|+|8-7.5|}{6} =$$

$$\frac{5.5 + 5.5 + 3.5 + 0.5 + 2.5 + 0.5}{6} = \frac{18.0}{6} = 3 \quad (2)$$

This indicates that there is less dispersion in group 1 than in group 4, as it should.

A somewhat similar measurement, called the standard deviation, denotes the root mean square of the deviations from the mean and is sometimes called the RMS deviation. The standard deviation of the numbers in the left-hand column of group 1 is obviously zero since each of the deviations from the mean is zero. However, the standard deviation for group 4 is

$$\sqrt{\frac{(2-7.5)^2 + (13-7.5)^2 + (4-7.5)^2 + (8-7.5)^2 + (10-7.5)^2 + (8-7.5)^2}{6}} =$$

$$\sqrt{\frac{79.50}{6}} = \sqrt{13.25} = 3.64 \tag{3}$$

The basic difference between the standard deviation and the mean deviation is the greater influence of extreme values on the standard deviation than on the mean deviation. If the nature of the problem is such that the influence of extreme values are deemed important, then the standard deviation is the preferable measure of dispersion. The square of the standard deviation is called the variance. For normally distributed data, the standard deviation and variance play a significant part in probability and statistical estimation theory.

<u>Applicability to terrain roughness</u>

To see how such measures might be used to describe terrain roughness, suppose a terrain elevation profile along a line is obtained and plotted in the continuous waveform as follows:

The dispersion measurements used previously will yield a number that describes the deviation of the elevation amplitudes from the mean value or from the level datum, if desired. This quantity should then provide a good measure of roughness. With this method, a large value of the standard deviation or variance indicates a large dispersion of the eleveations and, therefore, a large amount of terrain roughness; while a low value indicates a relatively smooth terrain. If the mean value is zero, then the RMS and standard deviation are the same, and the mean squared value and the variance are the same.

However, this means for describing terrain roughness is not quite adequate for evaluating the effect terrain roughness has on vehicle response because a vehicle is a finely tuned, frequency-dependent system with a different response for each different frequency of a sinusoidal displacement. This implies that a correlation of a vehicle's response to an input terrain requires a knowledge of the dispersion of the terrain amplitudes per unit frequency. This will yield a spectrum of dispersons which when summed will yield a single number representing the total dispersion. Such a spectrum, called a power spectral density function, serves the more flexible, dual purpose of describing terrain roughness in a manner that permits an evaluation of vehicle response. A cross-country terrain profile can be thought of as a random function of elevation values versus horizontal distance, i.e. there is no way to predict an exact elevation at some given distance. This randomness

becomes even more evident when consideration is given to the travel of a vehicle across an off-road area in which no specific path of travel is defined. In the same sense that a sinusoid can be defined by its amplitude and frequency, a random profile can be defined by its distribution of amplitudes and distribution of frequencies. Representative measurements have shown that the amplitudes of many terrain and road profiles can be reasonably approximated with a Gaussian or normal distribution.[7] Although such a distribution is not a stringent requirement for spectral analysis of profiles, it enhances the statistical analysis by permitting the use of the many concepts defined for normal distributions. Regarding the distribution of frequencies for most profiles, the height of the profile is approximately proportional to the wavelength of the frequency being considered. However, in determining a power spectral density from a profile, estimating the true function from a finite sample and determining the validity of the estimates are problems. A major consideration which is dealt with later in the text is the Weiner-Khintchine relation which relates the autocorrelation function and the power spectral density as Fourier transform pairs, and is valid only for random, stationary profiles.[2] Stationary means that the statistical properties of the profile do not change with position. While most profiles are not stationary in the strictest sense, valid estimates of power spectral density can still be made from profiles that are quasi-stationary.

The general problem of the study reported herein was to investigate the validity and practicality of applying spectral density methods and the inherent statistics to the problem of describing terrain roughness

and vehicle responses in terms of vehicle speed.

Objective

The specific objective of this study was to define and formulate the procedures for implementing a practical, compact description of terrain-vehicle speed (TVS) systems and thus establish a methodology for relating measures of ride to measures of terrain roughness. Using this methodology, reliable estimates can be made of the average speed a given vehicle can attain over given terrain as well as of its probabilities of exceeding certain specified levels of vibration.

This may provide a more practical means of including the "ride" problem in computerized analytical models used to quantify the terrain-vehicle-driver system, and thus provide the military with a reliable analytical tool for (a) developing more effective vehicles at less cost and (b) rating them on a competitive basis in accordance with the tasks to be performed.

Scope of Study

This study was entirely a computerized effort. Digital computer simulations were conducted in which a vehicle was run at several selected speeds over terrain profiles with various levels of roughness. The terrain profiles were generated by computer programs that constructed and shaped sequences of random normal numbers to provide representative profiles with the desired power spectrum, variance, mean value, standard deviation, and RMS. These programs also performed the necessary statistical checks for normality, stationarity, and randomness.

A two-dimensional, nonlinear mathematical model of an M37 truck served as the vehicle in these simulations. This model allowed for the nonlinearities inherent in large rotational motions, the suspension elements, bump stops, loss of ground contact, and the tire compliance, which was represented by a cluster of radially projecting springs.[6]

The analysis is presented chiefly in the form of graphs comparing input and output statistics and distribution functions. The input statistics consisted of terrain roughness as measured by RMS elevation. The output is represented as the RMS vertical acceleration of the vehicle's center of gravity.

A family of curves was developed relating vehicle response to terrain roughness for each speed of vehicle traversal. Cross plots of these established relations provided a more useful relation between vehicle response and speed for various degrees of terrain roughness.

CHAPTER II: DESCRIPTION OF COMPUTER CALCULATIONS

Digital Simulation of Low-Pass Gaussian Profiles

An essential requirement for this study was the capability to generate artificial terrain profiles composed of random, normally distributed amplitudes with a given degree of stationarity and control of the frequency content and pertinent statistics. Specifically, the first aim was to construct a group of terrain profiles that were different deterministically but were the same statistically, and then to systematically vary certain statistics.

A series of computer programs and subroutines, appropriately interwoven, generated these low-pass, Gaussian profiles with a zero mean and specified standard deviation by the following procedures: A random-number chain, generated by the power residue method, was entered at an arbitrary starting point, and 12 uniform, random numbers were computed and summed. By the central limit theorem, a single random, normal number was then computed by the formula:

$$V = (A - 6)(\sigma_n)$$

where

V = the random, normal number

A = the sum of 12 uniform, random numbers

σ_n = the standard deviation desired for the sequence of random, normal numbers.

This technique for computing random, normal numbers is often referred to as the "sum of the uniform deviates" method.[5]

Generally, the calculations were repeated until a sequence of 1000 random, normal numbers was obtained. Although the mean of the resulting sequence was always very nearly zero, a shifting operation was performed to insure a mean value of absolute zero. The frequency distribution of the sequence at this point approximates that of white Gaussian noise in that its spectral density is uniformly distributed over all frequencies with an average mean square level of σ_n^2 . To obtain the desired spectrum, the sequence was then passed through a numerical system that simulated an analog low-pass filter with a certain "time constant" $T = RC$ and cutoff frequency α .[1] The analog equivalent of this system is shown in Fig. 1. This system is formulated numerically by the following formula:

$$Y_{i+1} = X_{i+1}\,(0,\sigma_N) + Y_i e^{-\alpha\tau}$$

where

$X(0,\sigma_N)$ = sequence of random, normal numbers previously calculated

α = the system's cutoff frequency (temporal or spatial)

τ = the time (or distance) interval between points in the sequence

(NOTE: The points are assumed to be equidistant.)

Y = the resulting sequence.

The $\alpha\tau$ product gives complete control over the spectrum shaping. It was determined after several trials that $\alpha\tau \approx 0.055$ gave the best normality condition. The desired density level is achieved by specifying the appropriate standard deviation σ of the resulting sequence. This value is obtained by the formula:

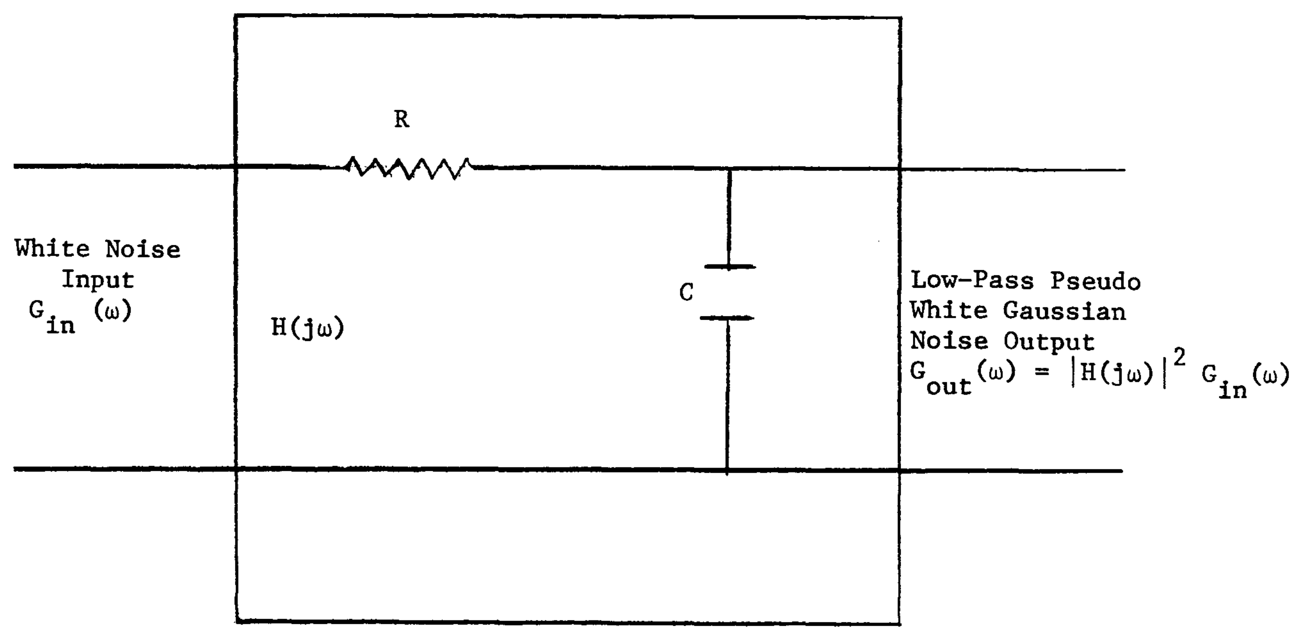


Figure 1. Low Pass RC Filter

$$\sigma = \frac{\sigma_N}{\sqrt{1 - e^{-2\alpha\tau}}}$$

The computer program entitled "NOISE," listed in Appendix A, performs the operations for generating the random, normal sequences.

Tests of Fundamental Assumptions

Tests for stationarity

The sequence of numbers representing a random terrain profile was divided into N equal intervals, where the data in each interval were considered to be independent of those in any of the other intervals. The mean and mean square values were computed for each interval. The resulting sequences of the mean and mean square values were tested for underlying trends by two tests, the run test and the trend test.[3] Generally, the run test is more powerful for detecting fluctuating trends, and the trend test is more powerful for detecting underlying monotonic trends. Stationarity required that the sample pass both the run and the trend tests at the 95 percent confidence level. The sequence of mean square values were each assumed to be independent samples of a random variable with a true mean μ and mean square value ψ^2. If this hypothesis were true, the variations in the sequence of sampled values would be random and display no trends. Hence, the number of runs in the sequence relative to some value (the mean) would be as expected for a sequence of independent random observations of the random variable, as given in Table 1. Moreover, the number of reverse arrangements in the sequence would be as expected for a sequence of

Table 1

Percentage Points of Run Distribution

n = N/2	α 0.99	0.975	0.95	0.05	0.025	0.01
5	2	2	3	8	9	9
6	2	3	3	10	10	11
7	3	3	4	11	12	12
8	4	4	5	12	13	13
9	4	5	6	13	14	15
10	5	6	6	15	15	16
11	6	7	7	16	16	17
12	7	7	8	17	18	18
13	7	8	9	18	19	20
14	8	9	10	19	20	21
15	9	10	11	20	21	22
16	10	11	11	22	22	23
18	11	12	13	24	25	26
20	13	14	15	26	27	28
25	17	18	19	32	33	34
30	21	22	24	37	39	40
35	25	27	28	43	44	46
40	30	31	33	48	50	51
45	34	36	37	54	55	57
50	38	40	42	59	61	63
55	43	45	46	65	66	68
60	47	49	51	70	72	74
65	52	54	56	75	77	79
70	56	58	60	81	83	85
75	61	63	65	86	88	90
80	65	68	70	91	93	96
85	70	72	74	97	99	101
90	74	77	79	102	104	107
95	79	82	84	107	109	112
100	84	86	88	113	115	117

independent random observations of the same variable, as given in Table 2. If the number of runs or reverse arrangements was significantly different from the expected number given in the tables, the hypothesis of stationarity would be rejected. The details of these two tests are presented here to give an understanding of the principles involved in the calculations.

Run test. Consider the sequences of mean values X_i and mean square values Y_i for a given profile to be classified into mutually exclusive categories identified by plus (+) or minus (-), where $X_i > \overline{X} = +$, $X_i < \overline{X} = -$; and likewise for the Y_i . A run is defined as a sequence of identical observations. The number of runs that occur in a sequence of observations indicates whether or not results are independent random observations.

Let it be hypothesized that there is no trend in the sequences, that is, no variations in the mean and mean square values, which implies stationarity. If the sequences of mean and mean square values are independent observations and the number of (+) observations equals the number of (-) observations, then the number of runs in the sequence will have a sampling distribution as given in Table 1. The hypothesis can be tested to any desired level of significance α by comparing the observed runs in the interval between $r_{n;\,1-\alpha/2}$ and $r_{n;\,\alpha/2}$, where $n = N/2$, N being the total number of observations. If the observed runs fall outside the interval, the hypothesis is rejected at the α level of

Table 2

Percentage Points of Reverse Arrangement Distribution

	α					
N	0.99	0.975	0.95	0.05	0.025	0.01
10	9	11	13	31	33	35
12	16	18	21	44	47	49
14	24	27	30	60	63	66
16	34	38	41	78	81	85
18	45	50	54	98	102	107
20	59	64	69	120	125	130
30	152	162	171	263	272	282
40	290	305	319	460	474	435
50	473	495	514	710	729	751
60	702	731	756	1013	1038	1064
70	977	1014	1045	1369	1400	1437
80	1299	1344	1382	1777	1815	1360
90	1668	1721	1766	2238	2283	2336
100	2083	2145	2198	2751	2804	2865

significance. For an example, consider the sequence of the observations below:

(1)	5.2	(6)	5.7	(11)	6.2	(16)	5.4
(2)	5.1	(7)	5.0	(12)	6.4	(17)	6.4
(3)	5.7	(8)	6.5	(13)	4.7	(18)	5.8
(4)	5.2	(9)	5.4	(14)	5.4	(19)	6.7
(5)	4.3	(10)	5.8	(15)	5.9	(20)	5.2

The mean value of the observations is 5.6. Let all observations with a value greater than 5.6 be (+) and those less than 5.6 be (-). The result is

$$\underbrace{--}_{1}\quad \underset{2}{+}\quad \underbrace{--}_{3}\quad \underset{4}{+}\quad \underset{5}{-}\quad \underset{6}{+}\quad \underset{7}{-}\quad \underbrace{+++}_{8}\quad \underbrace{--}_{9}\quad \underset{10}{+}\quad \underset{11}{-}\quad \underbrace{+++}_{12}\quad \underset{13}{-}$$

There are 13 runs and 20 observations. Let it be hypothesized that the observations are independent. The acceptance region for this hypothesis is

$$r_{10;\ 1-\alpha/2} < r < r_{10;\ \alpha/2}$$

From Table 1 for $\alpha = 0.05$, $r_{10;\ 1-\alpha/2} = r_{10;\ 0.975} = 6$, and $r_{10;\ \alpha/2} = r_{10;\ 0.025} = 15$. The hypothesis is accepted because 13(r) falls between 6 and 15. Therefore, the run test is passed and there is no evidence of an underlying fluctuating trend.

<u>Trend test</u>. The trend test counts the number of times that $x_i > x_j$ for $i < j$. Each such inequality is called a reverse arrangement. Let the total number of reverse arrangements be denoted by A. Test

the same sequence of 20 observations used in the previous example for a trend at the $\alpha = 0.05$ level of significance. For example, the first number, 5.2, is greater than four of the subsequent numbers. The number of reverse arrangements would therefore be 4. The second number, 5.1, is greater than three of the subsequent numbers, with the first number excluded from this comparison. The number of reverse arrangements is 3. The third number, 5.7, is greater than eight of the subsequent numbers, with the first two numbers excluded. Thus, the number of reverse arrangements is as follows:

$A_1 = 4$	$A_6 = 6$	$A_{11} = 6$	$A_{16} = 1$
$A_2 = 3$	$A_7 = 1$	$A_{12} = 6$	$A_{17} = 2$
$A_3 = 8$	$A_8 = 10$	$A_{13} = 0$	$A_{18} = 1$
$A_4 = 3$	$A_9 = 2$	$A_{14} = 1$	$A_{19} = 1$
$A_5 = 0$	$A_{10} = 4$	$A_{15} = 3$	

The total number of reverse arrangements is 62. Now let it be hypothesized that the observations are independent observations where there is no trend. The acceptance region for this hypothesis is

$$A_{20;\ 1 - \alpha/2} < A < A_{20;\ \alpha/2}$$

From Table 2 for $\alpha = 0.05$, $A_{20;\ 1 - \alpha/2} = A_{20;\ 0.975} = 64$ and $A_{20;\ \alpha/2} = A_{20;\ 0.025} = 125$. Hence, since $A = 62$ falls outside this interval, the hypothesis is rejected at the 5 percent level of significance. A plot of the 20 observations (Fig. 2) shows a slight monotonic increasing trend that was not detected by the run test. The straight line delineating the trend was determined by a least squares fit of the data. The sensitivity of the test at this level of significance is

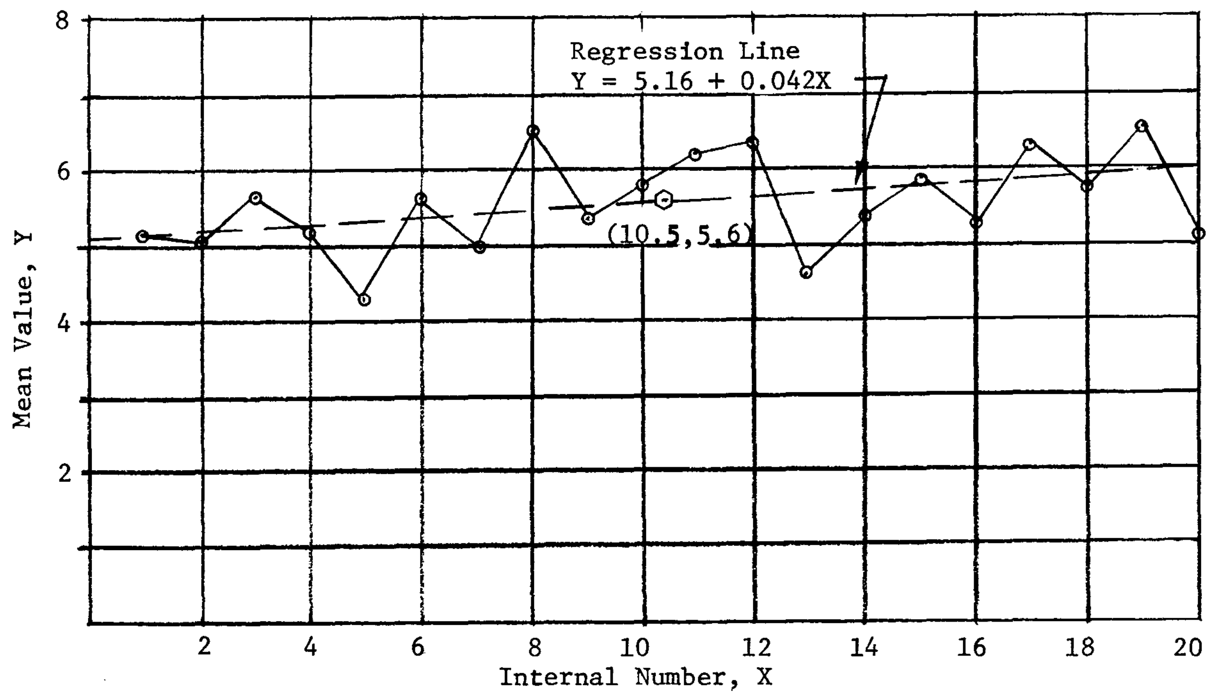


Figure 2. Mean Value Versus Interval Number

perceived from the small value of the slope of the regression line. Therefore, for a sequence such as this, the hypothesis of stationarity would be rejected.

Test for randomness

Randomness, the most fundamental concept underlying all the theory of estimation and testing of hypotheses, is far from easy to understand. It is generally interpreted as a lack of bias, implying no preference for any single element or group of elements. The determination of this lack of bias is centered around certain specific elements of bias (non-randomness) that can be detected from an analysis of the internal structure of the sample. This can be accomplished by recording the individual measurements that make up a sample in the precise order in which they were originally obtained and performing the run test used previously to test stationarity. However, this method is not practical (or necessary) for samples containing a large number of points. Having passed the test for stationarity, a test for randomness effectively reduces to a test for the presence of sinusoids due to periodic or almost periodic components in the data.[4] The most powerful method of detecting sinusoidal components in otherwise random data is presented by an autocorrelogram, i.e. a plot of the autocorrelation function. For any purely random data, the autocorrelation function will always approach a value equal to the square of the mean value as the lag becomes large. On the other hand, the autocorrelation function for a sine wave, or collection of sine waves, will continue to oscillate no matter how large the lag becomes. Details concerning the autocorrelation function and its computation are presented later in this report.

Test for normality

Normality is tested conveniently by the "Chi-square Goodness-of-the-Fit-Test." The general procedure of the test involves the use of a statistic with an approximate Chi-square distribution as a measure of the discrepancy between an observed probability density function and the theoretical probability density function. The approximate Chi-square value of the given data is obtained by:

$$X^2 = \sum_{i=1}^{K} \frac{(f_i - F_i)^2}{F_i}$$

Where f_i is the frequency of occurrence in the subgroup interval of the original sequence, K is the total number of subgroups into which the sequence is divided, and F_i is the number of expected occurrences of the theoretically normal distribution. The Chi-square goodness-of-fit is influenced by the choice of the number of subgroups, K . For the case where the test will be performed at $\alpha = 0.05$ level of significance, William[9] suggests that the minimum number of subgroups K be determined by the formula:

$$K = 1.87(N - 1)^{2/5} \tag{4}$$

where N is the total number of points in the sequence (1000 for this study). To visualize the magnitude of K for a given N , a limited tabulation is as follows:

N	K
100	12
200	16
300	18
400	21
500	22
600	24
700	26
800	27
900	28
1000	30
.	|
.	|
.	|
1500	35
.	|
.	|
.	|
2000	39

The region of acceptance at the $\alpha = 0.05$ level of significance is

$$\chi^2 < \chi^2_{n:\alpha}$$

where $\chi^2_{n:\alpha}$ is the Chi-square value of the theoretically normal distribution function at the significance level α.

A computer subroutine program was developed to test the normality of sequences of any length by means of this Chi-square concept. This program is a part of the STANOR program (Appendix A) that performs all the tests of fundamental assumptions previously mentioned.

Autocorrelation Function

Generally, the principal application for an autocorrelation function measurement of physical data is to establish the dependence of values upon one another. However, the main interest here is that the autocorrelation function affords a means of computing the power spectral density function. The autocorrelation function is found by taking two

identical waveforms, repeatedly shifting one with respect to the other, and then calculating the correlation coefficient for each shift. The correlation coefficient for a given shift or lag is obtained by multiplying the ordinates of the two waveforms at corresponding points on the horizontal scale and averaging the products over a sufficiently long range, usually several periods of the lowest frequency present, to provide a steady average. This is illustrated in Fig. 3a. The products $a_1 b_1$, $a_2 b_2$, $a_3 b_3$, etc., are computed, added together, then divided by the total number of products in the sample. The correlation function is obtained from a plot of the correlation coefficient as a function of the shift or lagged values (Fig. 3b). The shift values may be in terms of time or distance, depending upon whether the original waveforms are functions of time or distance. The autocorrelation function has a maximum value for zero shift, which is equal to the mean square value of the waveform, and decreases to zero, if the mean value of the waveform is assumed to be zero, for infinite value of the shift unless the waveform contains periodic components. If the waveform has periodic components, the autocorrelation functions also has periodic components of the same period. The autocorrelation functions for several common waveforms are shown in Fig. 3c.

The autocorrelation function of a discrete sequence is defined by

$$R_\chi(\tau) = \lim_{N\to\infty} \frac{1}{N} \sum_{k=1}^{N} \chi(k) \cdot \chi(k + \tau) \tag{5}$$

where $R_\chi(\tau)$ is the value of the autocorrelation function when the lagged distance is τ and $\chi(k)$ is the original sequence. The discrete form of equation 5 is a direct translation of the autocorrelation

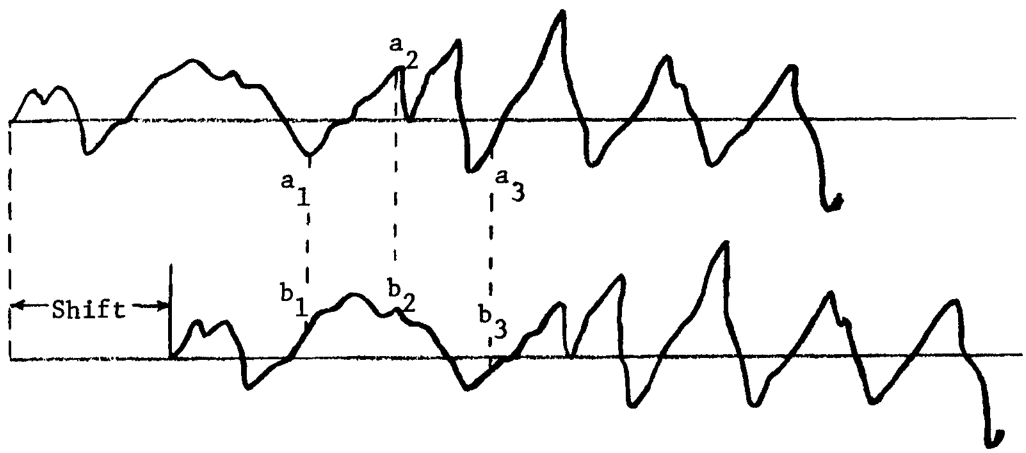


Figure 3a. Calculation of Correlation Coefficient

At corresponding points on horizontal scale, the ordinates of the two waveforms are multiplied. The correlation coefficient is the average of the $a_i b_i$ products.

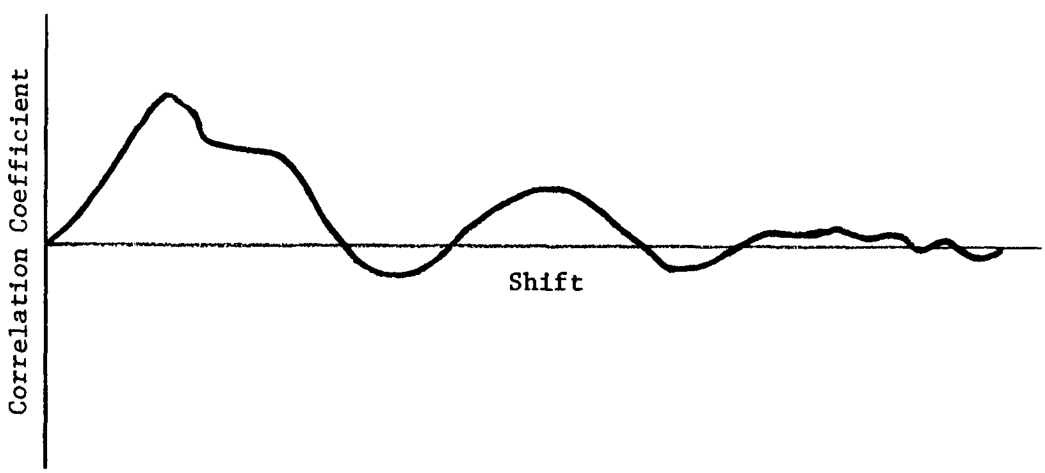


Figure 3b. The Correlation Function

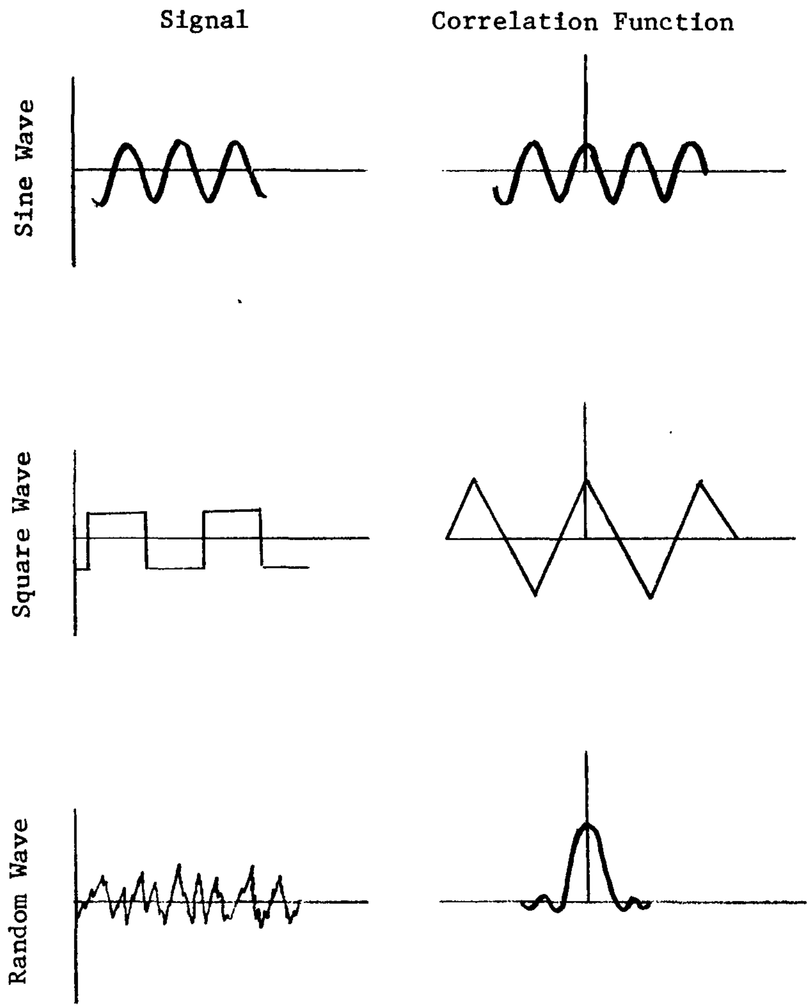


Figure 3c. Correlation Functions of Several Common Waveforms

function for a continuous and infinite sequence given by equation 6:

$$R_\chi(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_0^T \chi(t)\ \chi(t + \tau)dt \tag{6}$$

This equation is true for a sequence that has an infinite length. However, for sequences with finite length, consistent results require that the autocorrelation function (equation 5) be modified to the form

$$R_\chi(\tau) = \frac{1}{N-\tau} \sum_{k=1}^{N-\tau} \chi(k) \cdot \chi(k + \tau) \tag{7}$$

This equation differs from equation 5 in that the quantity $\chi(k) \cdot \chi(k + \tau)$ is averaged by dividing by the actual length over which $\chi(k)$ and $\chi(k + \tau)$ are multiplied.

For a random sample with a zero mean, this function should approach zero and remain at zero as the lag values increase, and the function should not be periodic. (See Fig. 3c for random wave.) The extent to which the function fulfills these conditions indicates whether the waveform (terrain profile) is sufficiently random to be described by its power spectral density. However, an unsatisfactory autocorrelation function can frequently be improved by performing a filtering operation (detrending) on the original data to remove the influence of longer wavelengths. A most useful and very significant property of the autocorrelation function is that it happens to be the Fourier transform of the power density spectrum of a random, stationary waveform. Therefore, having once obtained the autocorrelation function, it is then possible to compute the power spectral density function.

Power Spectral Density

The power spectral density (PSD) is a second moment or variance density spectrum. PSD of a random, stationary function is defined as the Fourier transform of its autocorrelation function as follows:

$$P(\Omega) = 2 \int_{-\infty}^{\infty} R_x(\tau)\, e^{-i2\pi\Omega\tau} d\tau \tag{8}$$

where $R_x(\tau)$ is the value of the correlation function for shift τ and Ω is the frequency in cyc/in. if τ is measured in inches and cyc/sec if measured in seconds. The integral over the frequency range $0 \to \infty$ of the PSD is the variance.* The raw estimate of the power spectrum is obtained by the following numerical method:

$$P_x^{(h)} = \frac{2\Delta t}{\pi} \sum_{\tau=0}^{m} \varepsilon_\tau R_x^{(\tau)} \cos \frac{h\tau\pi}{m} \qquad h = 0,1, \ldots m$$

where

$$\varepsilon_\tau = \begin{cases} 1 & 0 < \tau < m \\ 1/2 & t = 0,m \end{cases}$$

$R_x^{(\tau)}$ = autocorrelation value of series x (the waveform) at lag τ

$P_x^{(h)}$ = raw power spectral estimate of series x at frequency $\frac{h\pi}{m\Delta t}$

---

* This is true if the function has a zero mean. It was shown previously that for functions with a zero mean, the mean square and variance are the same.

NOTE: $P_x^{(h)} = P_x\left(\frac{h\pi}{m\Delta t}\right)$

Δt = constant sampling interval (may be time or distance)

m = maximum lag used in autocorrelation function not to exceed 199.

The raw estimates are then smoothed by using "Hamming" coefficients as follows:

$$SP_x^{(0)} = 0.54\ P_x^{(0)} + 0.46\ P_x^{(1)}$$

$$SP_x^{(h)} = 0.23\ P_x^{(h-1)} + 0.54\ P_x^{(h)} + 0.23\ P_x^{(h+1)},\quad 0 < h < m$$

$$SP_x^{(m)} = 0.54\ P_x^{(m)} + 0.46\ P_x^{(m-1)}$$

where $SP_x^{(h)}$ is the smoothed power spectral estimate of series x at frequency $\frac{h\pi}{m\Delta t}$. Smoothing is necessary because the raw estimate is an inefficient estimate of the true spectral density because the finite record length precludes identifying frequencies exactly. A more reliable statistical estimate can be obtained by averaging over neighboring frequencies. Trial-and-error methods have proved more powerful in obtaining sets of averaging coefficients that produce reliable spectral estimates than has any particular theory. There are several well-established sets of coefficients available, and in general, the Hamming method is quite satisfactory. The computer program "PSD" (Appendix A) computes both the autocorrelation and the PSD estimates.

A plot of two terrain power spectra is shown in Fig. 4. The ordinates are in terms of ft^2/cyc/ft, and the abscissa are in units of cyc/ft, the reciprocal of wavelength. These spectra measure the rate of change of the mean squared elevation at each frequency with frequency and are primarily geometric quantities that do not contain any

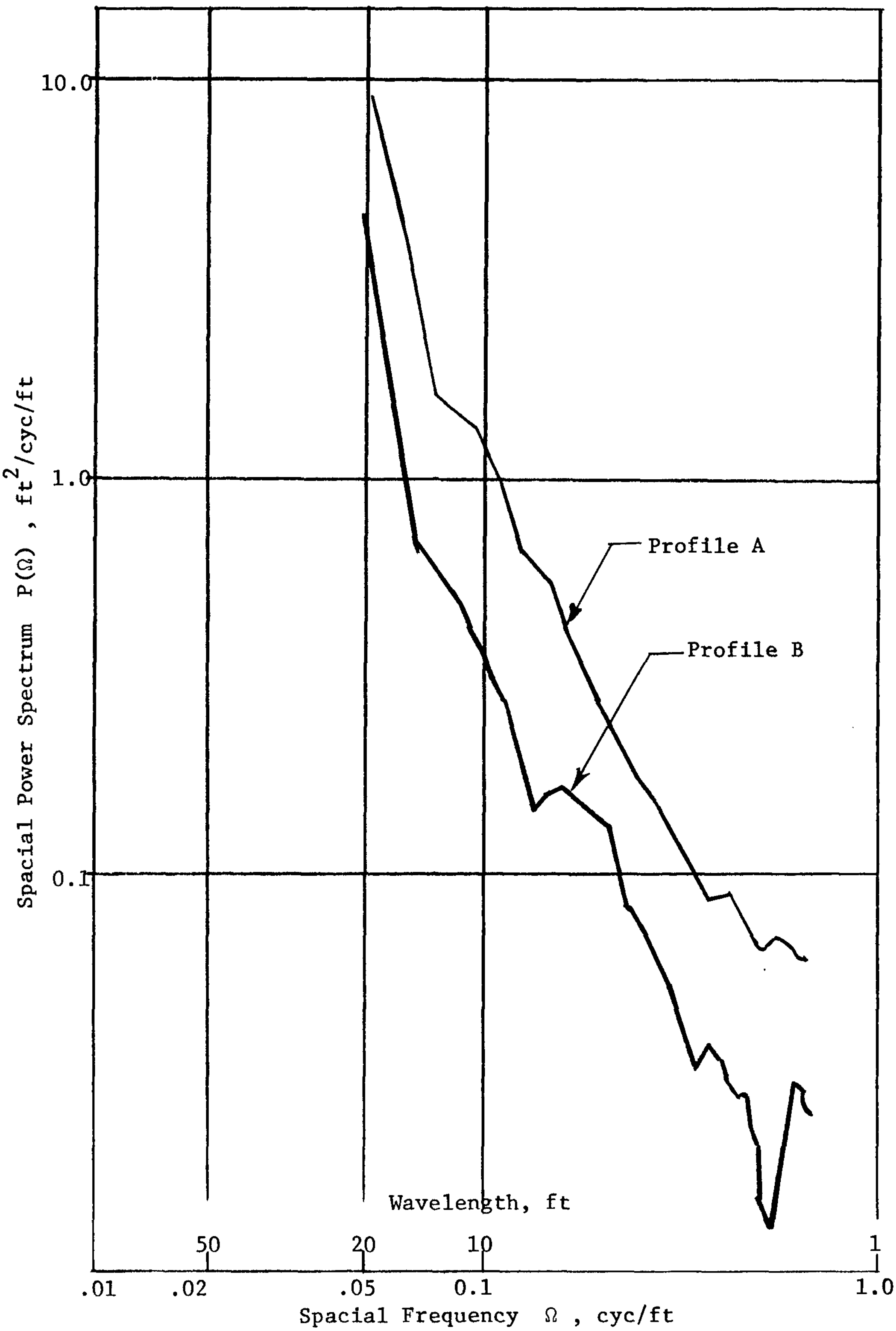


Figure 4. Examples of Terrain Power Spectra

units of time. The area under each power spectrum represents the mean square value of the total roughness for the respective profile. The area under the curve between any two selected wavelengths will indicate the contribution to the total roughness that this range of wavelengths produces. Thus, from the terrain power spectrum curve, the wavelengths that make significant contributions to the variation in the elevation measurements can be determined.

The terrain elevation power spectrum can be used also with certain vehicle characteristics to determine wavelengths and frequencies that induce significant forces and motions if the velocity of the vehicle is introduced. This is done by multiplying the abscissa of the power spectrum curve by the vehicle velocity to obtain the frequency cycles per second (cps), and by dividing the ordinates of the power spectrum by the vehicle velocity to obtain the units of ft^2/cyc/sec. A different curve obviously results for each velocity because changes in velocity tend to stretch or shorten the curve. Now, take a hypothetical case where a vehicle is driven onto a sinusoidal exciter platform with a controlled frequency, adjustable amplitude, and electronic transducers to measure the resulting reaction forces and motions. By varying only the frequency, a response of the force/unit displacement versus frequency (Fig. 5) can be obtained. Note the two predominant frequencies at about 2 and 15 cps. These are the basic natural frequencies of the center of gravity and the axles. Variations in the terrain profile that will excite frequencies of 2 and 15 cps will thus generate large forces and motions between the vehicle and the terrain. These frequencies are, therefore, of interest in determining the influence of

Figure 5. Force-Frequency Vehicle Response

the terrains on the reaction of the vehicle. This relation between forces (or motion if so desired) and frequency can now be combined with the terrain power spectrum to obtain an estimate of the dynamic force or motion that the vehicle will exert on the terrain, and vice versa. To do this, the terrain spectrum must first be modified by introducing the vehicle velocity, as previously described. If this is done, then to determine a power spectrum of the dynamic force that the vehicle will exert on the terrain is simply a matter of multiplying the two spectra. It is worth mentioning that the vehicle characteristics (Fig. 5) must be squared when this mathematical operation is performed to give the resulting units of lb^2/cyc/sec in the ordinates of the response spectrum. A typical result of these computations can be perceived from the plots in Figs. 6 and 7 for the 44 ft/sec speed. The influence of the vehicle's natural frequencies (2 and 15 cps) is vividly portrayed by the dynamic response spectrum (Fig. 7). This response spectrum also has the same characteristics as previous spectra, i.e. the area under the curve represents the mean square value of the response. With this value, it is easy to determine the root mean square (effective) value. This RMS response value is the effective force (or motion) that the terrain exerts on the vehicle or vice versa, for that velocity. Such a spectral analysis provides a great deal of information on and insight into just how the frequencies of the terrain influence a vehicle's response and further pinpoints critical speeds and terrain wavelengths. However, this analysis is based on the fact that the vehicle is a linear system; that is, if the amplitude of the sinusoidal shaker is doubled, the only effect on the vehicle's response is the doubling of the output

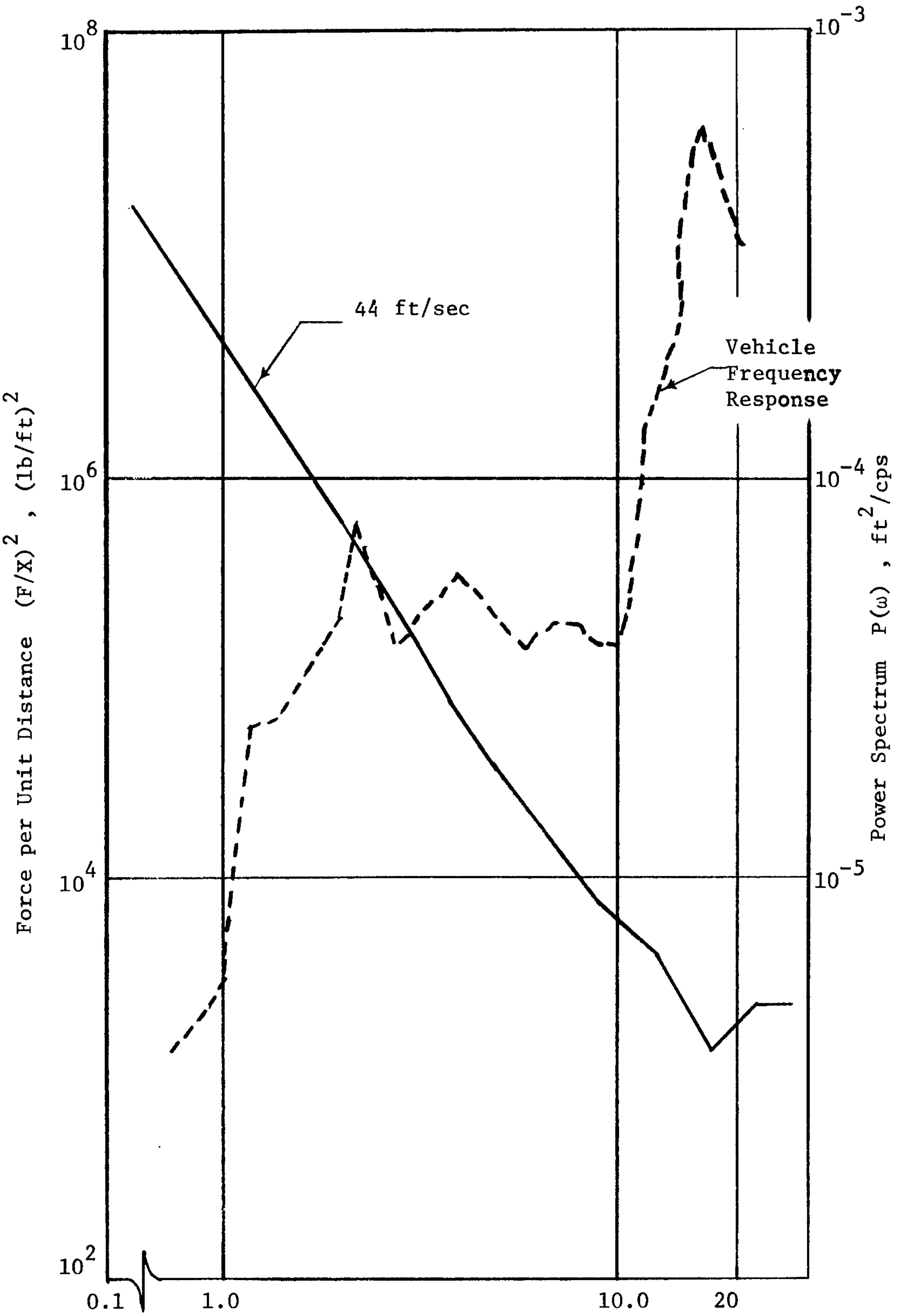


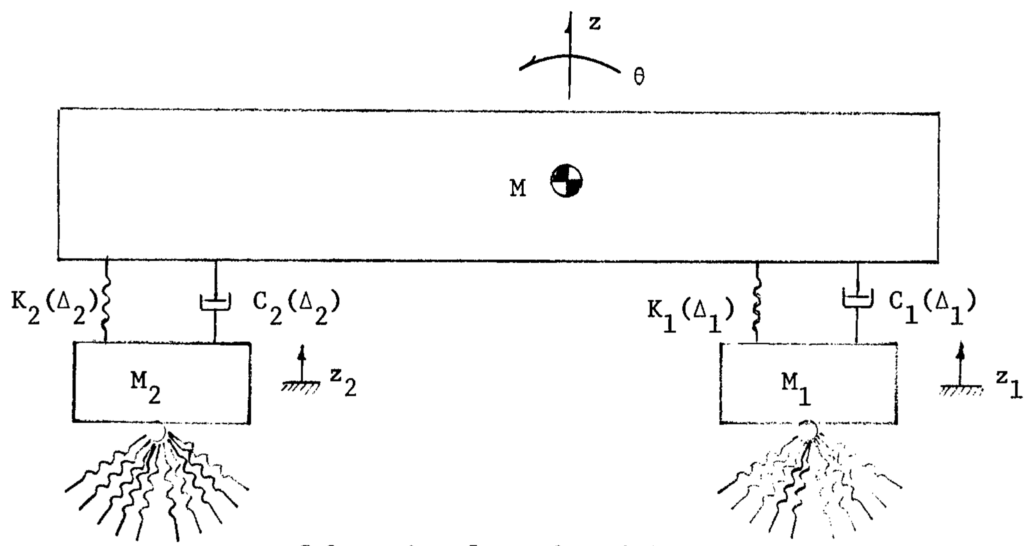
Figure 6. Terrain and Vehicle Characteristics as a Function of Frequency

Figure 7. Vehicle Response-Frequency Relation

for all frequencies. This, however, is not the case for actual vehicles which, unfortunately, are nonliner.

Vehicle Model

A two-dimensional, 4-deg-of-freedom mathematical model for the M37, 3/4-ton truck was developed for this study. The truck was modeled in terms of a series of coupled second-order ordinary differential equations that are solved simultaneously to describe the motions of the truck as it traverses the selected terrain profiles. The frame of the truck was considered rigid, and only the pneumatic tires and suspensions were considered to contribute to the sprung motion of the frame. The model included all the pertinent nonlinearities in the suspensions, those inherent in large rotational motions and loss of ground contact. Also, the segmented wheel concept[6] was employed to describe tire-terrain compliance. This added still another nonlinear feature to the model. The equations and a schematic diagram for the model are as follows:



Schematic of Truck Model

a. Forces on body.

$$M\ddot{z} = K_1(\Delta_1)(z_1 - z - a \sin \theta) + C_1(\dot{\Delta}_1)(\dot{z}_1 - \dot{z} - a\,\dot{\theta} \cos \theta)$$
$$+ K_2(\Delta_2)(z_2 - z + b \sin \theta) + C_2(\dot{\Delta}_2)(\dot{z}_2 - \dot{z} + b\,\dot{\theta}$$
$$\cos \theta) - Mg$$

$$I\ddot{\theta} = K_1(\Delta_1)a(z_1 - z - a \sin \theta) + C_1(\dot{\Delta}_1)a(\dot{z}_1 - \dot{z} - a\,\dot{\theta} \cos \dot{\theta})$$
$$- K_2(\Delta_2)b(z_2 - z + b \sin \theta) - C_2(\dot{\Delta}_2)b(\dot{z}_2 - \dot{z} + b\,\dot{\theta}$$
$$\cos \theta)$$

b. Forces on front axle.

$$M_1\ddot{z}_1 = - K_1(\Delta_1)(z_1 - z - a \sin \theta) - C_1(\dot{\Delta}_1)(\dot{z}_1 - \dot{z} - a\,\dot{\theta}$$
$$\cos \theta) + \sum_{i=1}^{10} \gamma_{i1}(p_{i1} - z_1) - M_1 g$$

c. Forces on rear axle.

$$M_2\ddot{z}_2 = - K_2(\Delta_2)(z_2 - z + b \sin \theta) - C_2(\dot{\Delta}_2)(\dot{z}_2 - \dot{z} + b\,\dot{\theta}$$
$$\cos \theta) + \sum_{i=1}^{10} \gamma_{i2}(p_{i2} - z_2) - M_2 g$$

where

$$\Delta_1 = z_1 - z - a \sin \theta \qquad \dot{\Delta}_1 = \dot{z}_1 - \dot{z} - a\,\dot{\theta} \cos \dot{\theta}$$
$$\Delta_2 = z_2 - z + b \sin \theta \qquad \dot{\Delta}_2 = \dot{z}_2 - \dot{z} + b\,\dot{\theta} \cos \theta$$

For this study, the suspension spring coefficients were presented by third-order polynomials, as shown in Fig. 8. These polynomials were obtained by curve fitting the actual force-deflection relations, and they are reasonable approximations. The suspension damping is as follows:

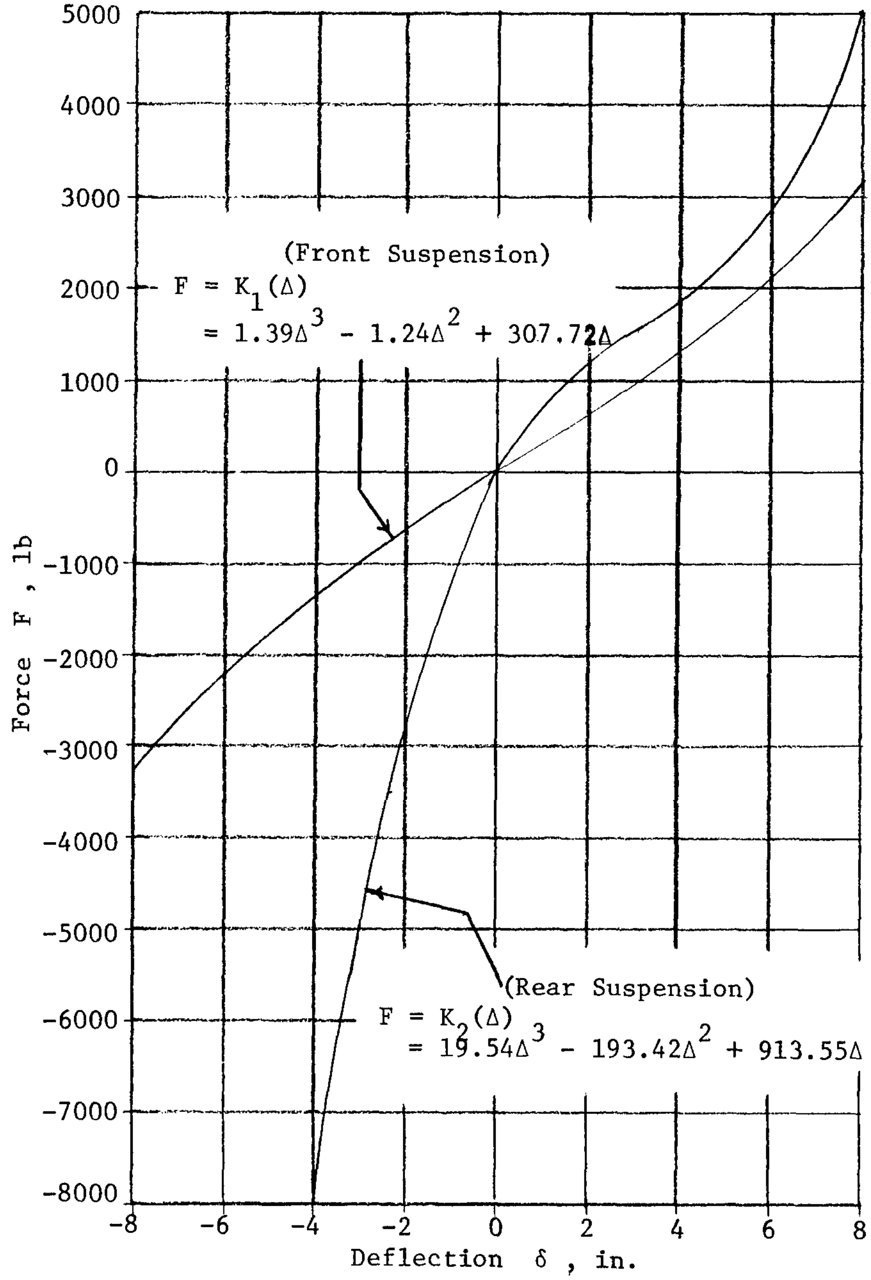


Figure 8. Force Versus Deflection for Front and Rear Suspensions of M37 Truck Model

$$C_1(\dot{\Delta}_1) = 11.8 \frac{\text{lb - sec}}{\text{in.}} \text{ (compression)}$$

$$= 22.8 \frac{\text{lb - sec}}{\text{in.}} \text{ (extension)}$$

$$C_2(\dot{\Delta}_2) = 12.0 \frac{\text{lb - sec}}{\text{in.}} \text{ (compression)}$$

$$= 49.0 \frac{\text{lb - sec}}{\text{in.}} \text{ (extension)}$$

No damping was incorporated in the tire compliance since in actual vehicles this damping is negligible compared to that of the suspensions. This truck model was forced to traverse each terrain condition at a constant speed, and the outputs consisted of motions of the main frame and axles in terms of displacement-, velocity-, and acceleration-time histories. The differential equations were programmed for solution on the GE-400 computer by the Runge-Kutta-Gill numerical integration scheme (Appendix A).

CHAPTER III: TEST PROCEDURES AND DATA ANALYSIS

Similarity of Input and Output Statistics

The response of linear systems to random excitations is well defined in the form of mathematical solutions employing the concept of transfer functions. For example, if a random input forcing function can be assumed to have a normal probability distribution about a mean value of zero, then the response of this input will also be a random quantity with a normal probability distribution about a mean value of zero. In other words, the input statistics (the terrain) are transformed in a linear fashion by the transfer characteristics of the system (the vehicle) to produce response statistics that are related by the expression

$$P(\omega)_{out} = |H(\omega)|^2 \, P(\omega)_{in}$$

where

$P(\omega)_{out}, P(\omega)_{in}$ = the output and input spectral density functions, respectively

$H(\omega)$ = the system transfer function.

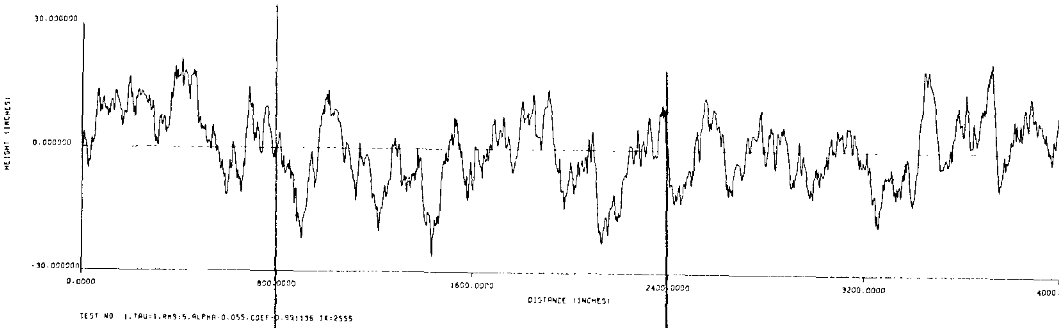
Hence, for a given input spectral density function, the output spectral density function is defined as the product of this spectral density function and the square of the system's transfer function.

Unfortunately, the topic of nonlinear response to random excitations does not readily lend itself to analytical treatment. Consequently, the most productive investigations in this area have been empirical in nature. Very little is known about how random inputs are transformed by nonlinear, multi-degree-of-freedom systems.

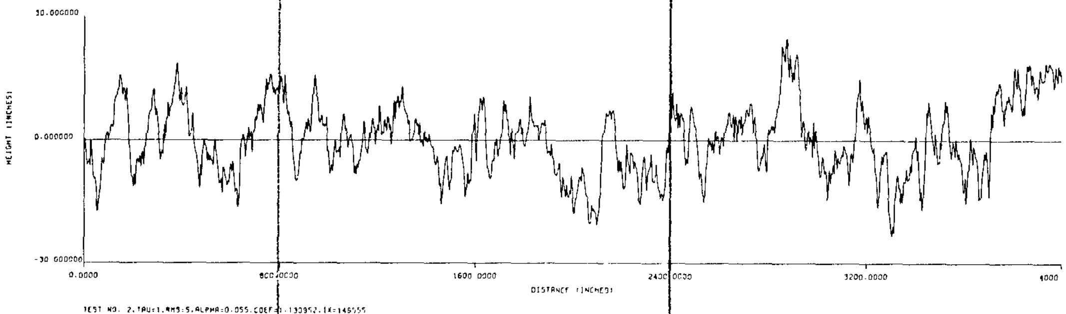
Before any effort was made to develop a method for defining terrain-vehicle-speed relations in a statistical fashion, an important question had to be answered: "Will constant statistical inputs to a nonlinear system produce constant statistical outputs?" A negative answer to this question is equivalent to saying that there is little hope for assembling a rational scheme for statistical analysis of non-linear systems. To answer this question, three terrain profiles were generated (Fig. 9) that were different deterministically, but were the same statistically. That is, they had essentially the same PSD, distribution function and identical RMS values, although the elevation sequences composing their waveforms were quite different. The construction of these profiles was influenced by the fact that a number of investigators have observed that the power spectra (PSD) of natural surfaces and most man-made surfaces, such as runways and highways, can be expressed by the equation:

$$P(\Omega) = C\Omega^{-n} \tag{9}$$

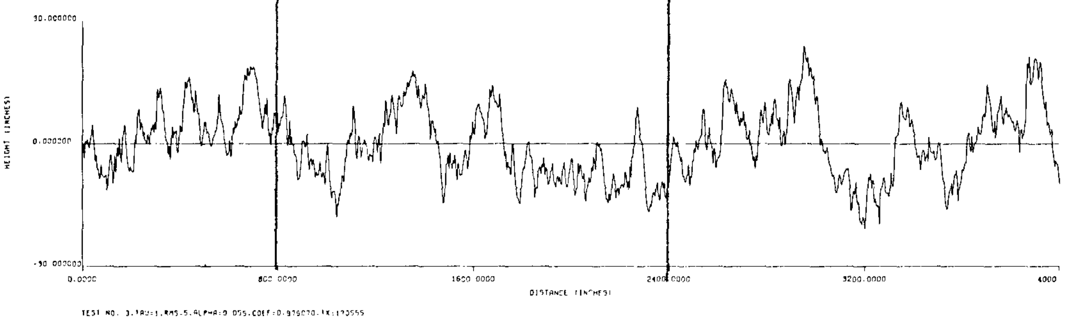
Published data further indicate that the exponent n for both natural and man-made surfaces is approximately two.[8] Therefore, a requirement for constructing these profiles was that their power spectra would be approximated by equation 9 with an exponent of two. Although the nearly 20-to-1 ratio between the vertical and horizontal scales produces distorted plots, the differences in features of the profiles are apparent. The RMS elevation was calculated for each profile, and in each case, a value of 8.5 was obtained. The RMS-distance plots are shown in Fig. 10. These profiles were purposely made extremely rough



Profile No. 1



Profile No. 2



Profile No. 3

Figure 9. Computer-Generated Profiles with Same Statistical Content

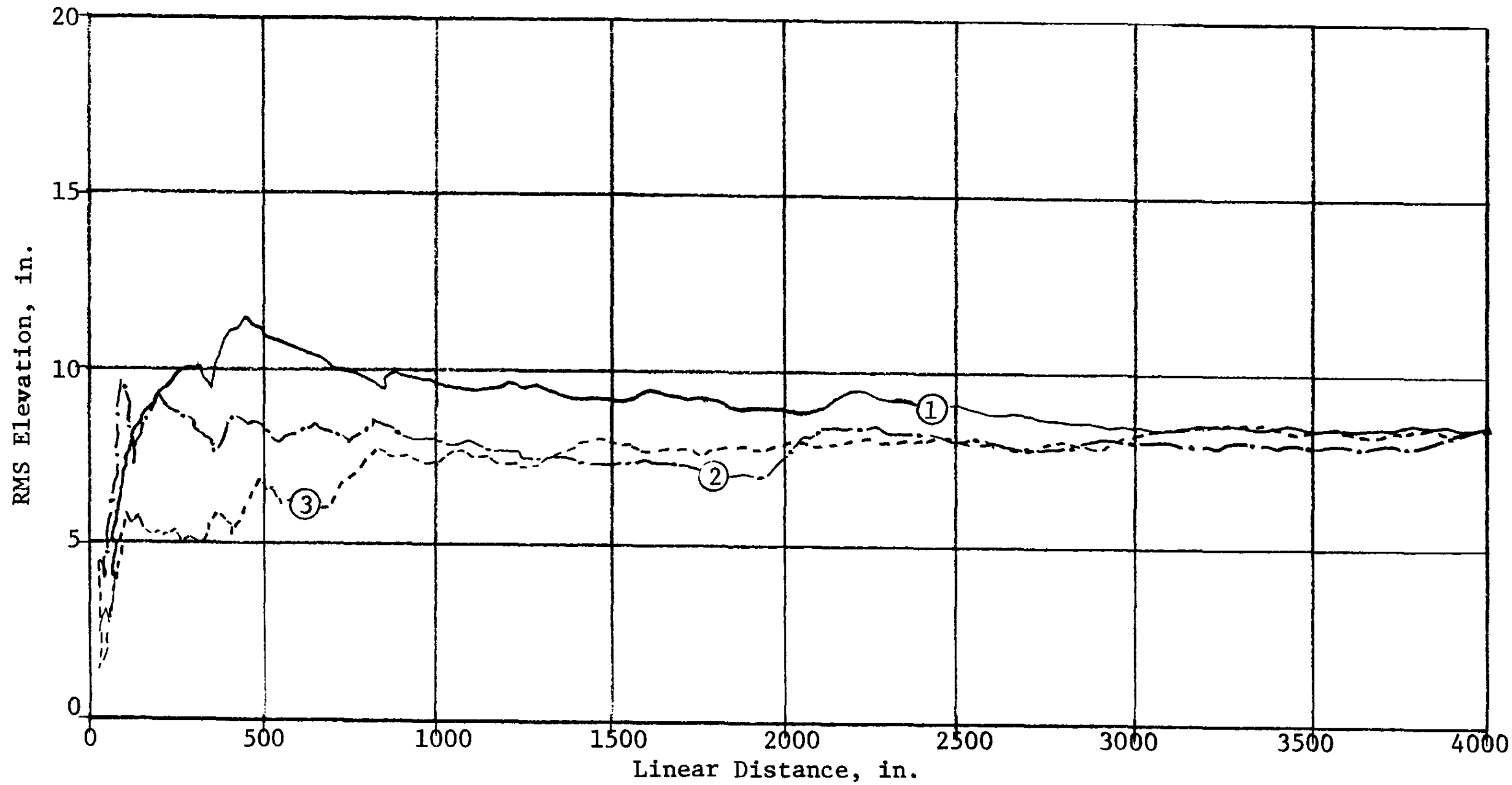
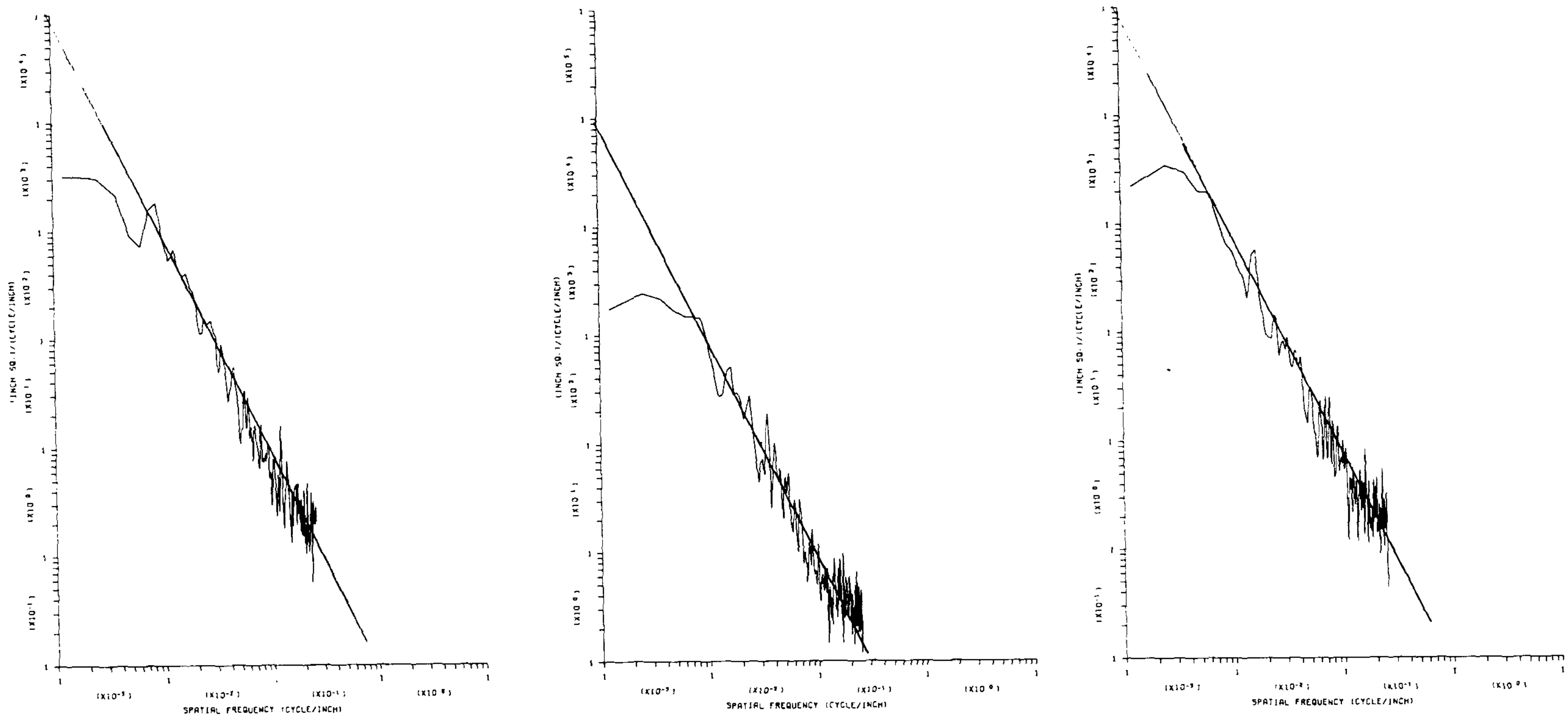


Figure 10. RMS Elevation Versus Distance for Computer-Generated Terrains

to assure that all the vehicle's nonlinearities would be influenced by the maximum degree. Logarithmic plots of the profile spectra are shown in Fig. 11. The straight line, whose equation is $P(\Omega) = 0.0785\Omega^{-2}$, drawn through each plot is to enhance a visual comparison of the similarity among the three spectra and illustrate the capability available for approximating given spectral relations.

The truck model traversed each of the three profiles at a speed of 5 mph (88 in./sec). The vertical acceleration-time histories of the vehicle's CG and front axle are shown in Figs. 12 and 13, respectively. Their respective power spectra are given in Figs. 14 and 15. The corresponding acceleration-time histories appear similar in form, but it is obvious from the distribution of the peaks that the waveforms are not truly stationary. However, if a quasi-stationary condition is considered for each case, the response spectra can be computed. Close examination of the corresponding response spectra reveal similar frequency distributions, but slight deviations in the low frequency range are noted for the CG motion. Unfortunately, at this speed this low frequency range corresponds closely to the basic natural frequency of the vehicle CG. Also, the axle response spectra for the test on terrain No. 3 is noticeably lower than that on the other two spectra. These deviations in the output spectra are reflected in the RMS acceleration-distance histories (Figs. 16 and 17). In Fig. 16, which shows the RMS-distance history of the CG motion, the RMS traces tend to converge and reach values of 1.02, 0.95, and 0.91 g, respectively, after about 4000 in. of travel. The RMS deviations at this point are really quite small, and indications are that they would converge to a common value



Profile No. 1 | Profile No. 2 | Profile No. 3

NOTE: The equation for the lines drawn on each plot is $P(\Omega) = 0.0785\Omega^{-2}$

Figure 11. Logarithmic Plots of the Profile Spectra for the Computer Generated Profiles with Similar Statistical Content

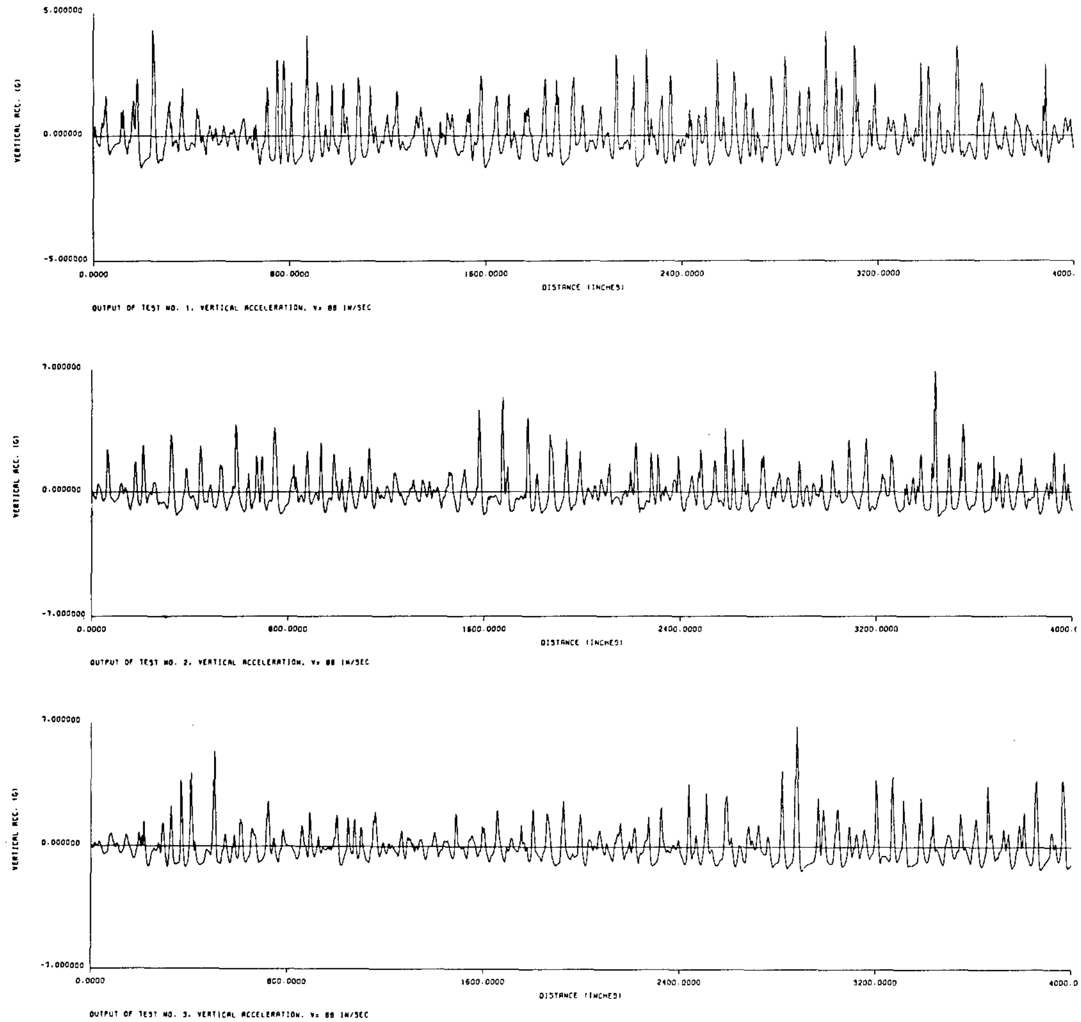


Figure 12. Acceleration-Time Histories of Vehicle's Center of Gravity Traversing Three Profiles with Same Statistical Content at 5 mph

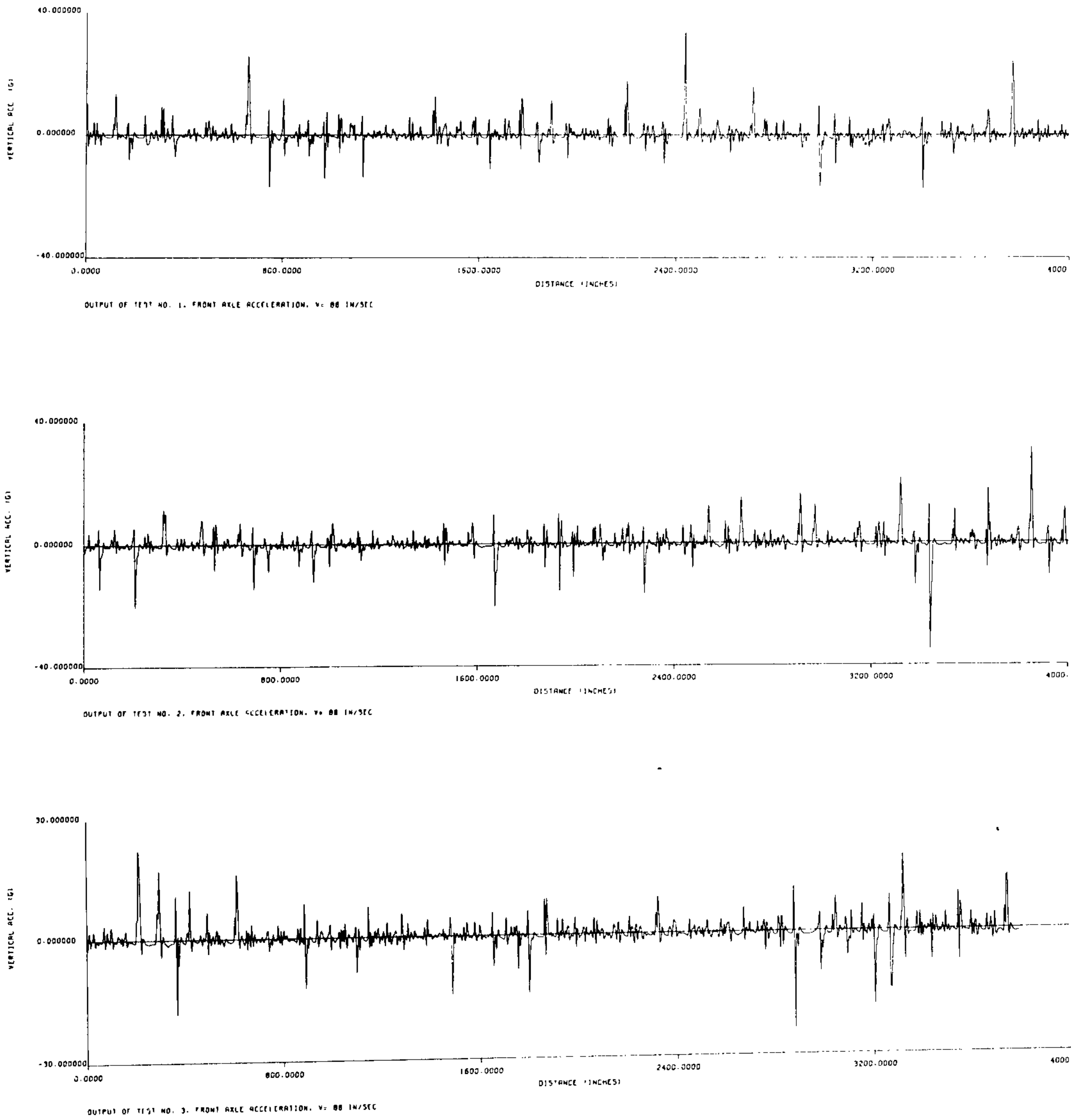


Figure 13. Acceleration-Time Histories of Vehicle's Front Axle Traversing Three Profiles with Same Statistical Content at 5 mph

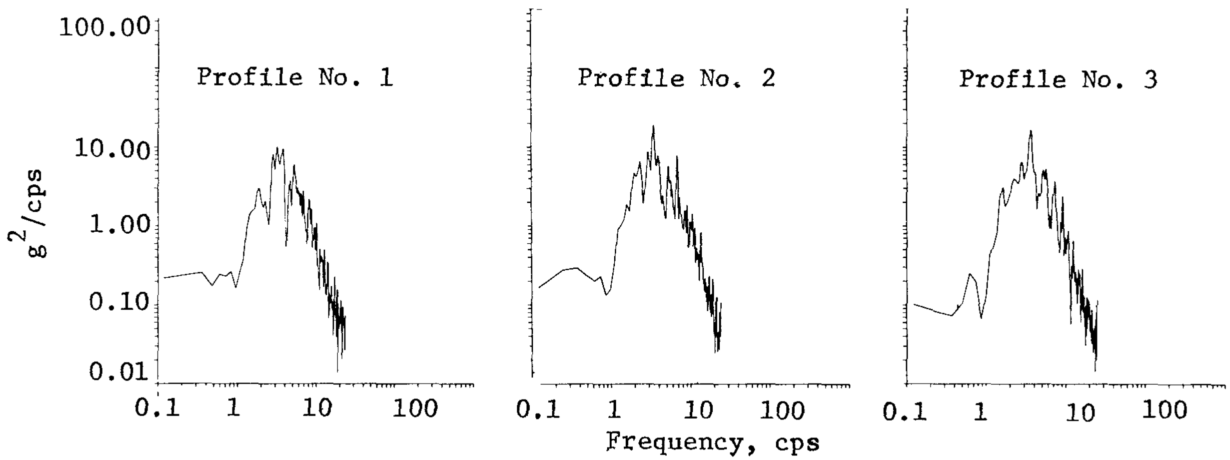


Figure 14. Response Spectra for CG Acceleration

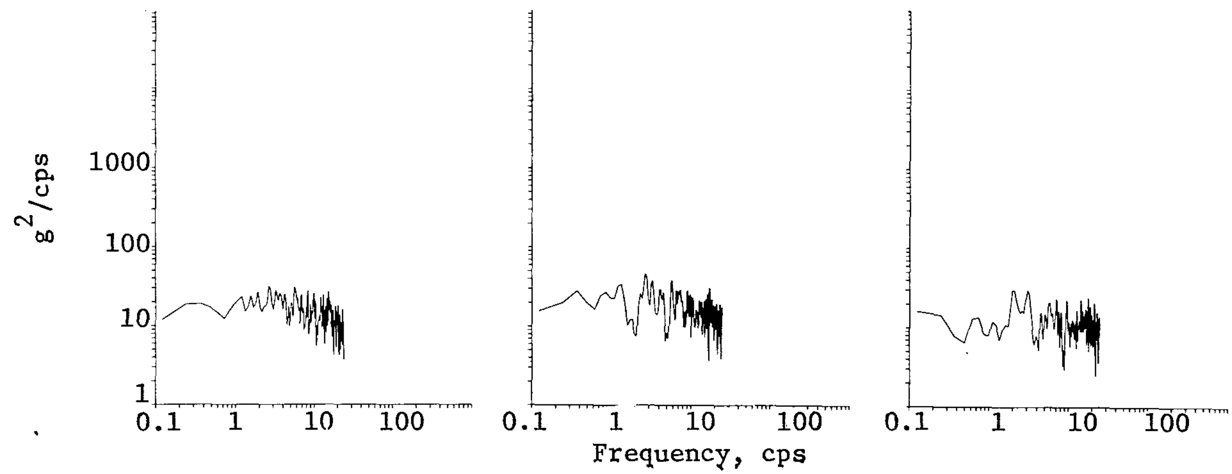


Figure 15. Response Spectra for Front Axle Acceleration

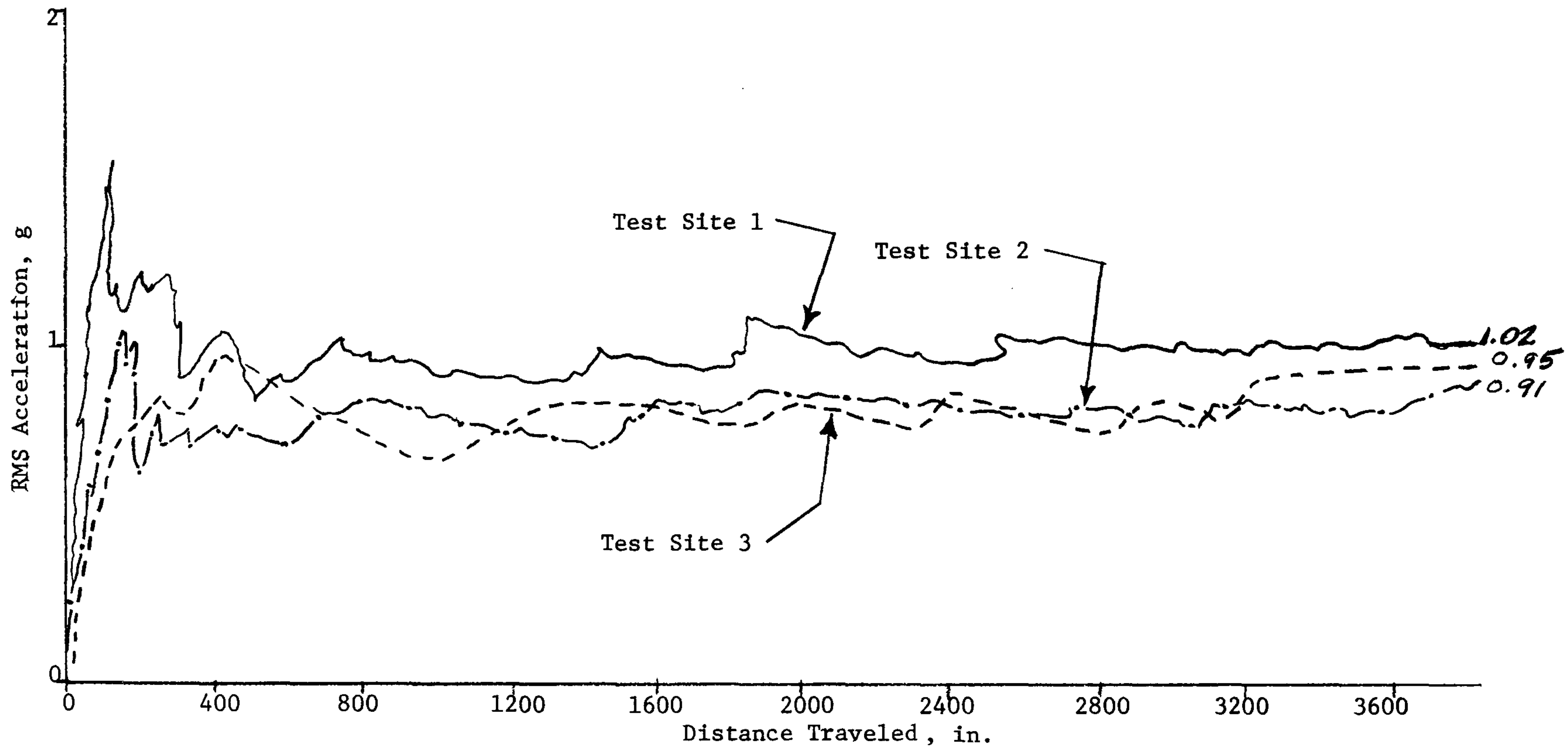


Figure 16. RMS Acceleration versus Distance of CG Response

if the test were evaluated over a longer distance.

A somewhat similar occurrence is noted for the axle motions (Fig. 17), where terminal RMS values of 3.55, 3.38, and 3.06 g were recorded. As indicated by the response spectra, the RMS acceleration at the axle for the test on terrain No. 3 was lower than that recorded for the other two. The noticeable deviations in the low frequency range of the CG response are believed to further indicate an insufficient length of sampling. Each of these tests represented only about 330 ft of travel for a vehicle with a wheel base of over 9 ft.

The autocorrelation functions of the CG motions were plotted for each of the three tests, and the results are shown in Fig. 18. In each test, a noticeable periodicity occurs. This is believed due largely to the periodic influence of the front and rear axles hitting the same profile elevations with a time delay dependent on the wheel base and the vehicle speed. This is probably the chief contributor to the nonstationarity of the output. Therefore, in all probability if the test had been conducted long enough to allow these periodic influences to achieve stationarity, it is conceivable that at this point and beyond the output RMS results would coincide. If this were the case and since this periodic phenomenon is a function of the wheel base, it appears logical that a minimum distance criterion could be established in terms of the vehicle's wheel base for evaluating vehicle responses in cross-country environments. For example, one might say that to statistically evaluate the dynamic response of cross-country vehicles requires travel over a profile that is suitably stationary and whose length is at least n times that of the wheel base of the

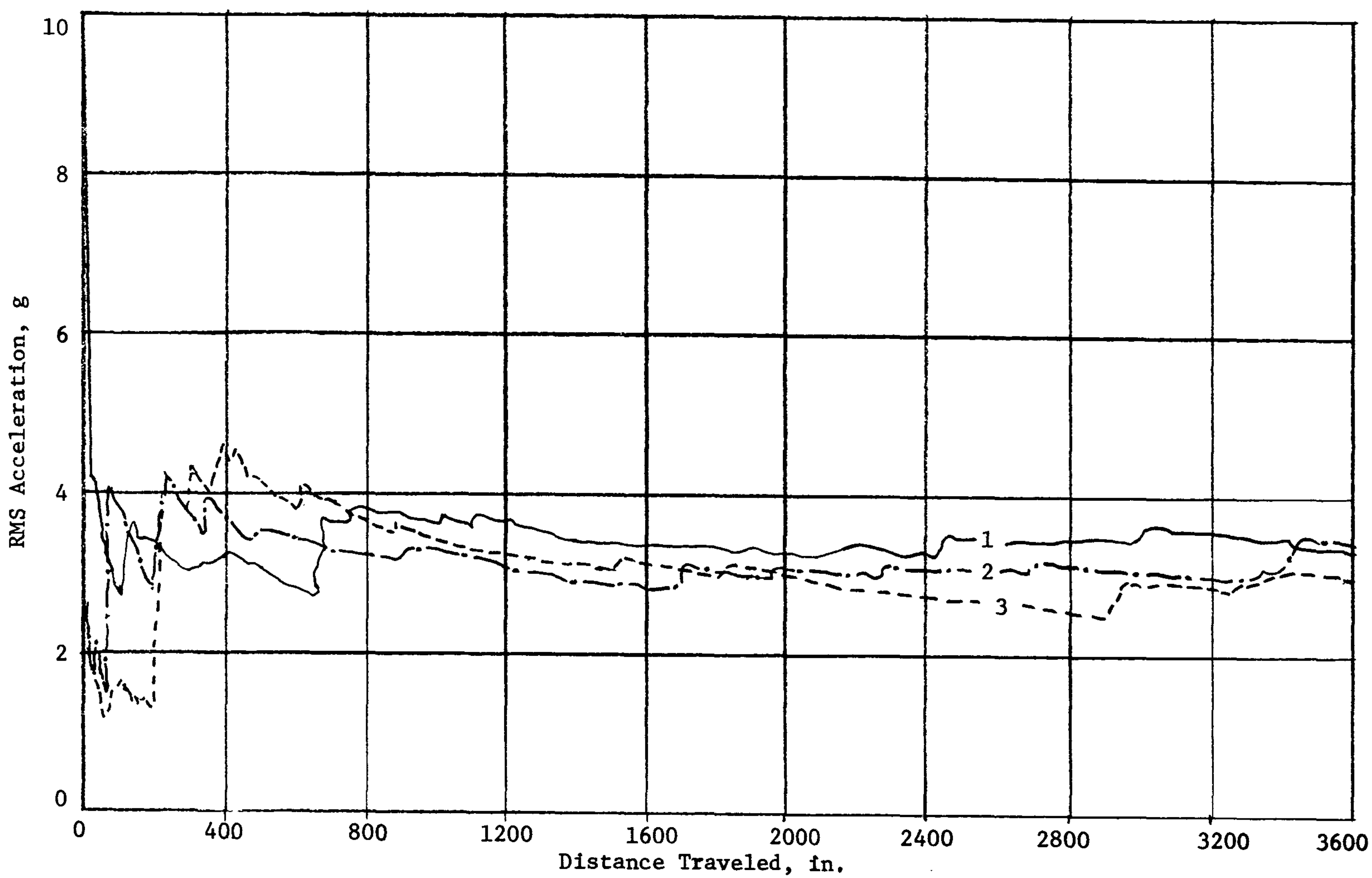


Figure 17. RMS Acceleration Versus Distance for Front Axle Motion

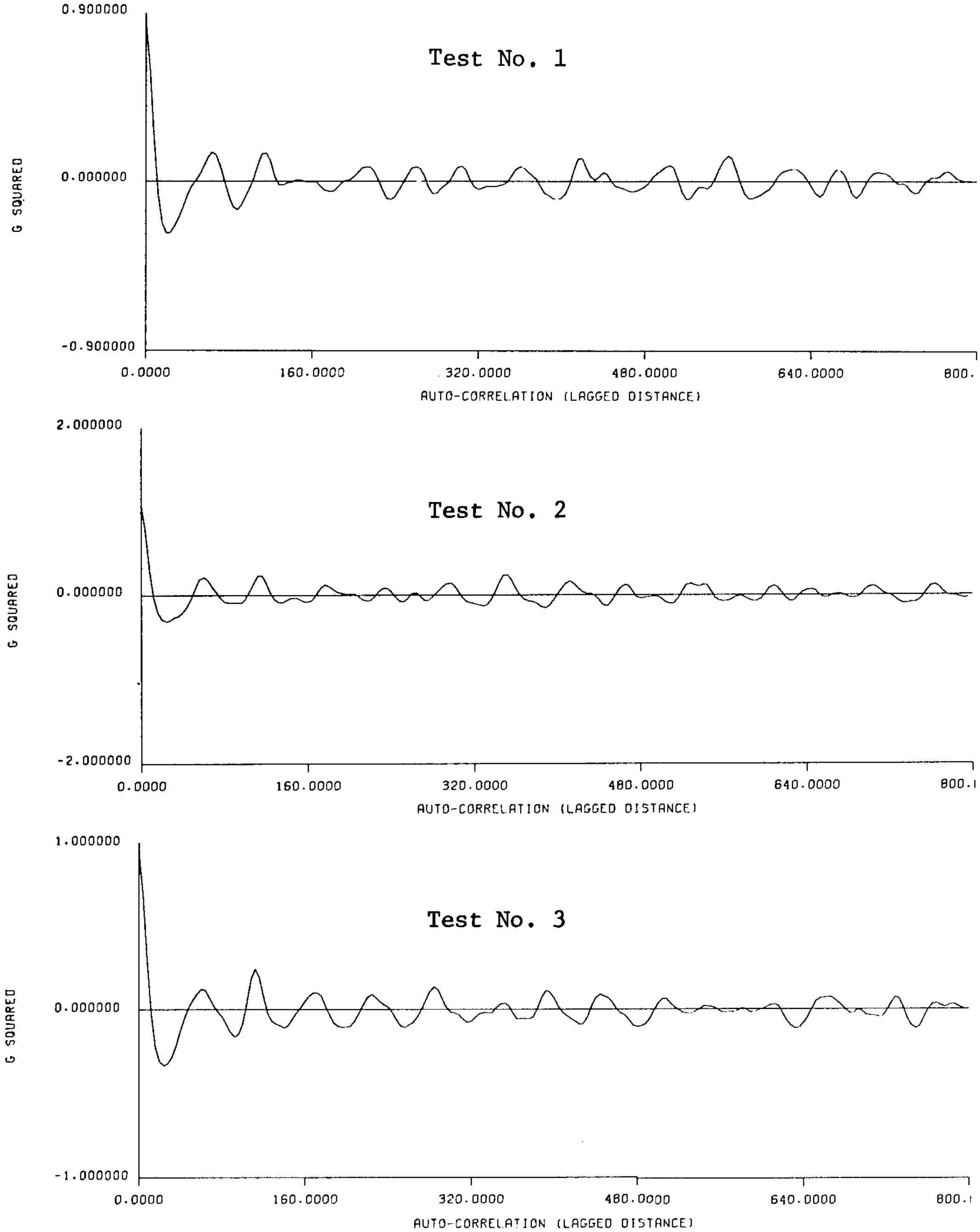


Figure 18. Autocorrelation Functions of Vehicle's CG Acceleration

vehicle. The value for n would have to be established from either experimental data or simulations.

Development of TVS Relations

It was concluded from the previous tests that the vehicle response statistics obtained from traversing terrains with constant statistics and a given terrain power spectra will be similar. This fact allows progression to the next step, which is the main objective in this study: to determine the effect of terrain roughness and vehicle speed on the vehicle's statistical response. Simulations with the truck model were conducted over eight profiles of varying levels of terrain roughness at speeds of 5, 7.5, 10, 15, and 20 mph over each of the eight profiles. Only the RMS elevation of the profile was varied, while the shape of the spectra was maintained constant. The nature of this variation is illustrated in Fig. 19, which shows logarithmic plots of the terrain profile spectra for three levels of roughness. The RMS elevations for these three profiles were 2.8, 8.5, and 63.7, respectively. The profile with RMS = 63.7 would preclude any type of vehicular traffic and is shown only to illustrate that the profile roughness can be changed without altering the profile's frequency distribution. This method of changing only the profile RMS simply alters each elevation point by a proportionate amount. That is, if the original profile had an RMS of 5 and the RMS was changed to 10, the features of the resulting profile would be identical to those of the original, but the elevations would be twice those of the original.

The straight lines drawn through the three spectra each represent

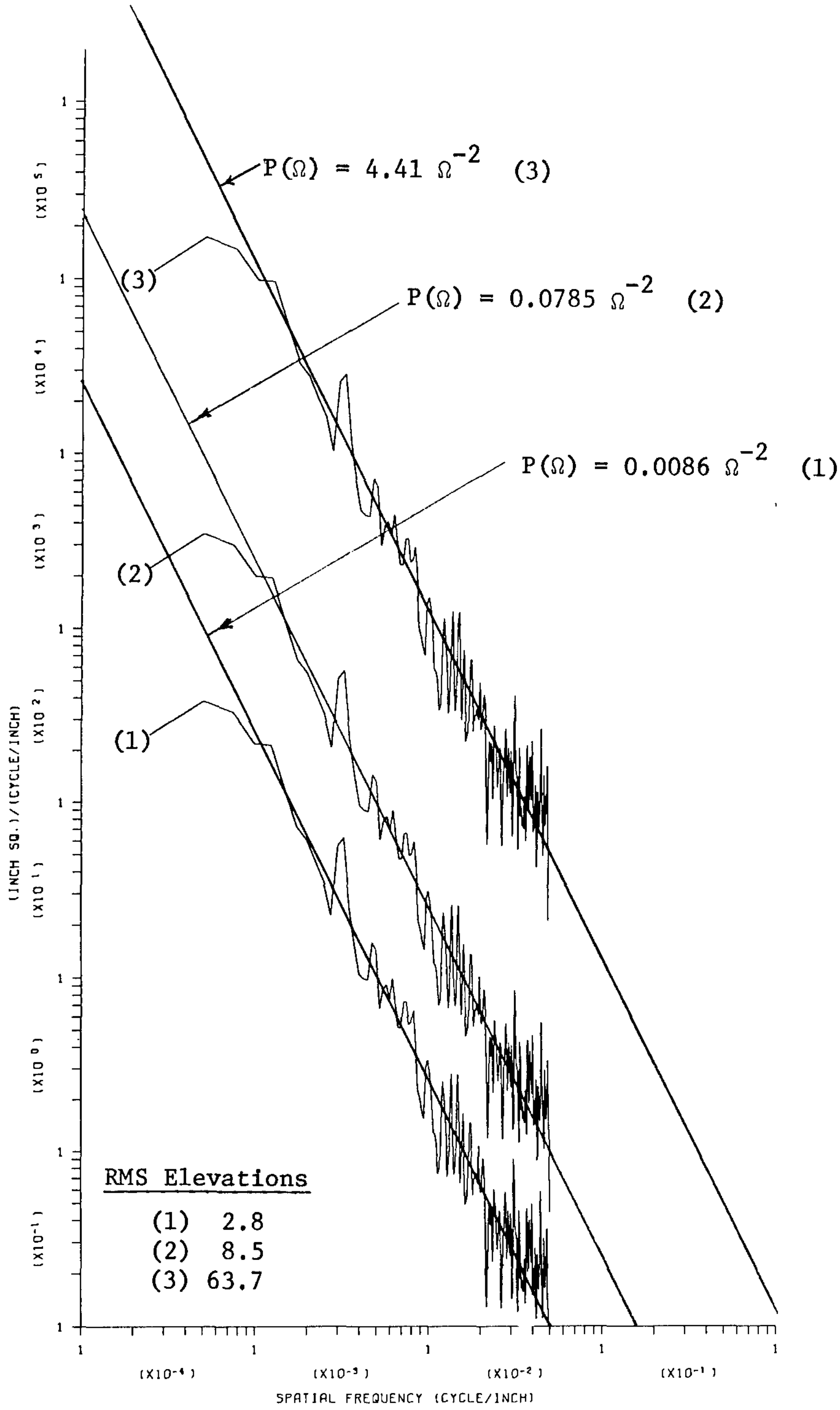


Figure 19. Power Spectra for Computer Generated Terrains of Three Levels of Roughness

a power function with a negative exponent of two. The coefficients of these power functions are related to the RMS by means of another power expression:

$$RMS = CA^{1/2}$$

where A is the coefficient for a particular spectra and C is a constant whose magnitude depends on the frequency range considered.

The results of the simulations are shown in Fig. 20 in the form of RMS acceleration of the vehicle's CG versus RMS roughness. There is a distinct response pattern for each speed, but the influence of the system's nonlinearities at the higher speeds on the rougher profiles caused pronounced deviations from a well-behaved pattern. This is typical of frequency-dependent systems; and, because of this, several more profiles and speeds should be inserted to adequately define the response pattern. To get some idea of the response sensitivity, an extra profile (RMS = 5.0) was included for the 20-mph speed and two (RMS = 4.5, 5.2) for the 10-mph speed. The most noticeable aspect of the additional profiles was a pronounced peak that occurred in the response pattern at 20 mph when the profile roughness changed from RMS = 4.9 to RMS = 5.0. This is undoubtedly due to the occurrence of resonant conditions that caused excessive contact between the axles and the bump stops and produced accelerations that would be harmful to the vehicle as well as its occupants in an actual situation. Perhaps for a more realistic range of speeds and terrain roughness the response-roughness patterns would all be well behaved.

Since speed and ride severity are of chief interest in cross-country ride dynamics, a more usable graph was obtained by cross

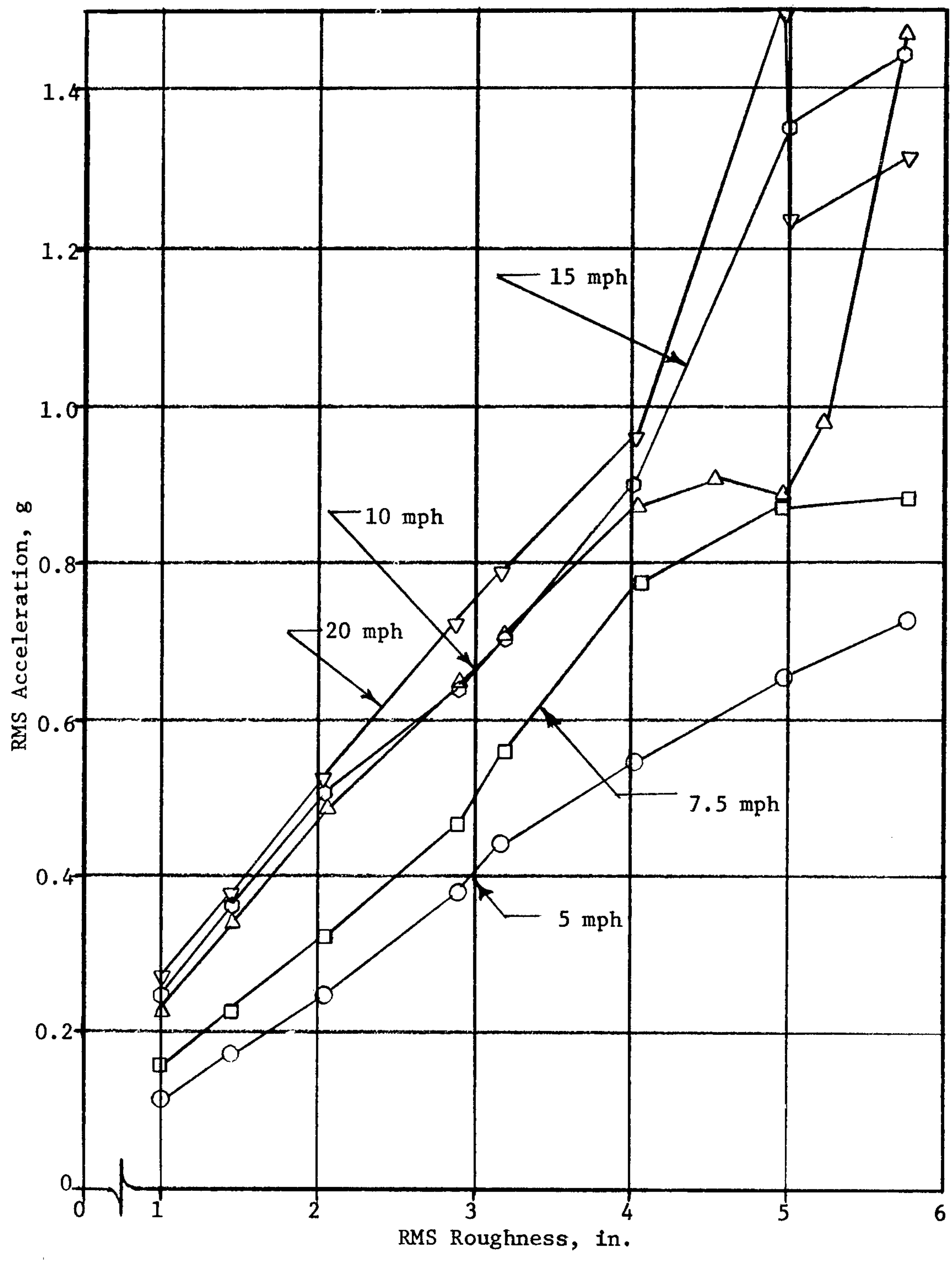


Figure 20. RMS Acceleration Versus RMS Terrain Roughness; Vertical Motion at Vehicle's CG; M37 Truck Model

plotting these results and relating the vehicle response to vehicle speed. These results are shown in Fig. 21. More tests are needed at speeds below 5 mph to adequately define the initial portions of the response-speed curves in Fig. 21. This range, shown by dashed lines, would most likely be the critical range for an actual TVS system, particularly for the rougher profiles. The dashed lines were made to intercept at a small common value at zero speed. This represents the small RMS level due to engine vibration that occurs in an actual vehicle when the vehicle is idle. These two graphs almost completely define the TVS characteristics for this particular vehicle model. For example, for a given level of ride severity based on RMS acceleration and a given level of terrain roughness based on RMS elevations, estimates of the average speed that the vehicle can attain and stay below the selected ride level can be made.

However, because of the statistical nature of the development, a complete definition of the TVS characteristics should provide a determination of the probability of exceeding the criteria of interest. This is attained by means of the amplitude probability distribution (APD), which is the probability that the amplitudes composing a waveform will exceed a given level. For digital data, an estimate of the APD is determined by simply counting the number of points above the given level and calculating the ratio of this number to the total number of available points.

For the terrain profile, the APD serves as a means for estimating the occurrence of elevations above a given level. For vehicle response, the APD serves to estimate the probability of exceeding a given g-level. Such a distribution function was constructed from the absolute values

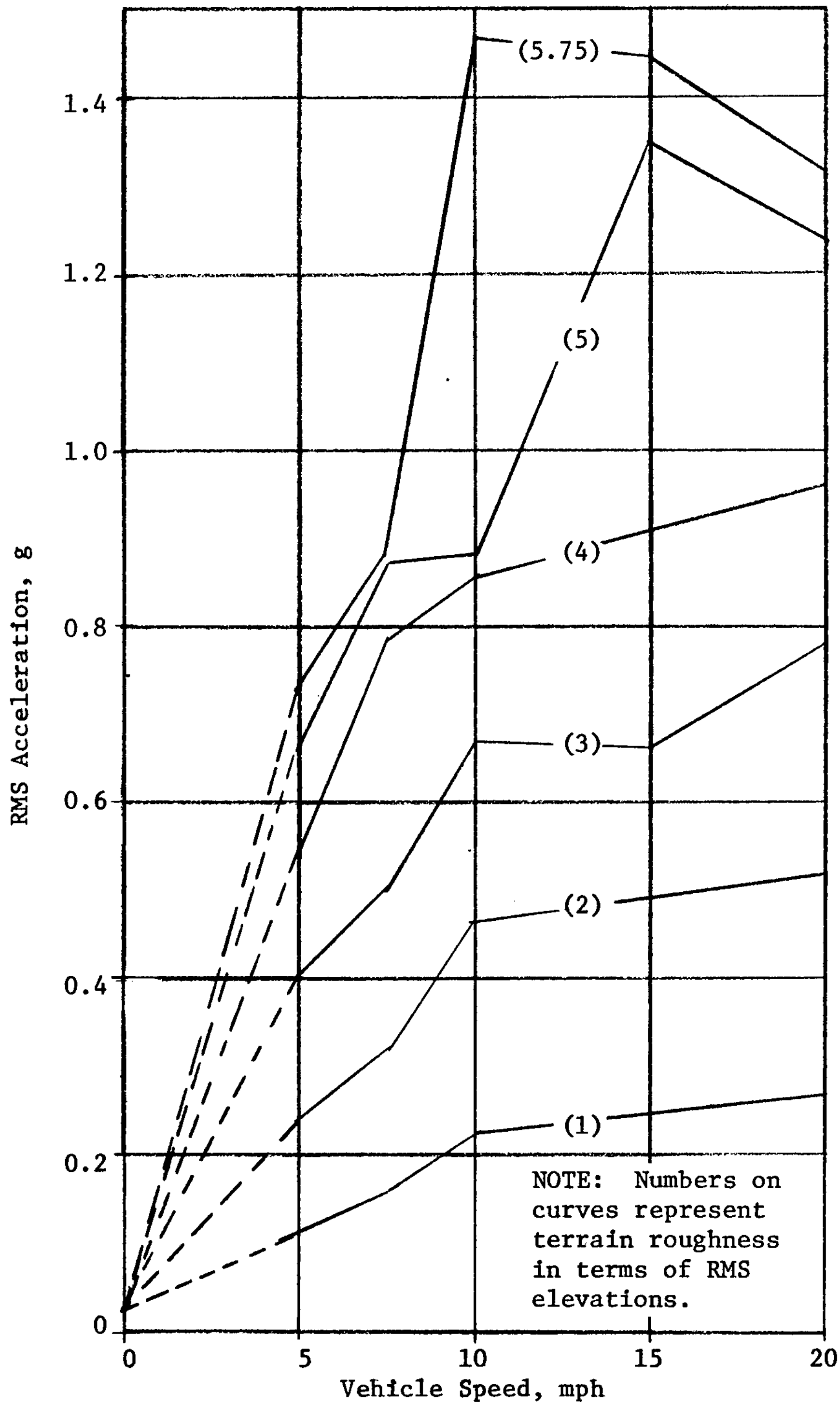


Figure 21. RMS Acceleration Versus Vehicle Speed; Vertical Motion at Vehicle's CG; M37 Truck Model

of the CG accelerations for the test at 7.5 mph over the terrain with an RMS roughness of five, the results are shown in Fig. 22. The maximum acceleration that occurred during this test was 3.85 g. According to amplitude distribution, the probability of exceeding 2.0 g is about 0.90, while the probability of exceeding 3.0 g is only about 0.26. Looked at in another way, when a vehicle is traversing a terrain of this nature at 7.5 mph, it can be expected to exceed 3 g for about 26 percent of the duration of the run. The response spectra will give the frequency of occurrence of these g levels. This leads quite naturally into a possibility for quantifying the intensity of a vibrational environment in terms of human tolerance. For example, such quantities as the frequency of occurrence of accelerations above a certain g level, or the area under the RMS acceleration-time history curve (which is a sort of impulse measure or measure of vibrational energy) should correlate with appropriate measures of human tolerance.

Figs. 20, 21, and 22 thus completely define the ride dynamics characteristics for the vehicle model operating on such random undulating profiles. However, since these profiles were constructed from a basic spectral shape that has been shown to represent most natural and man-made surfaces, it is conceivable that such TVS relations could be developed for any desired vehicle and serve to describe its ride dynamics characteristics for both on-road and off-road environments.

This graphical scheme will allow rating of vehicles, on a standard competitive basis of ride, as to their capability to negotiate cross-country terrains at various speeds. Such a comparison could take the form of the hypothetical relation in Fig. 23, which shows the

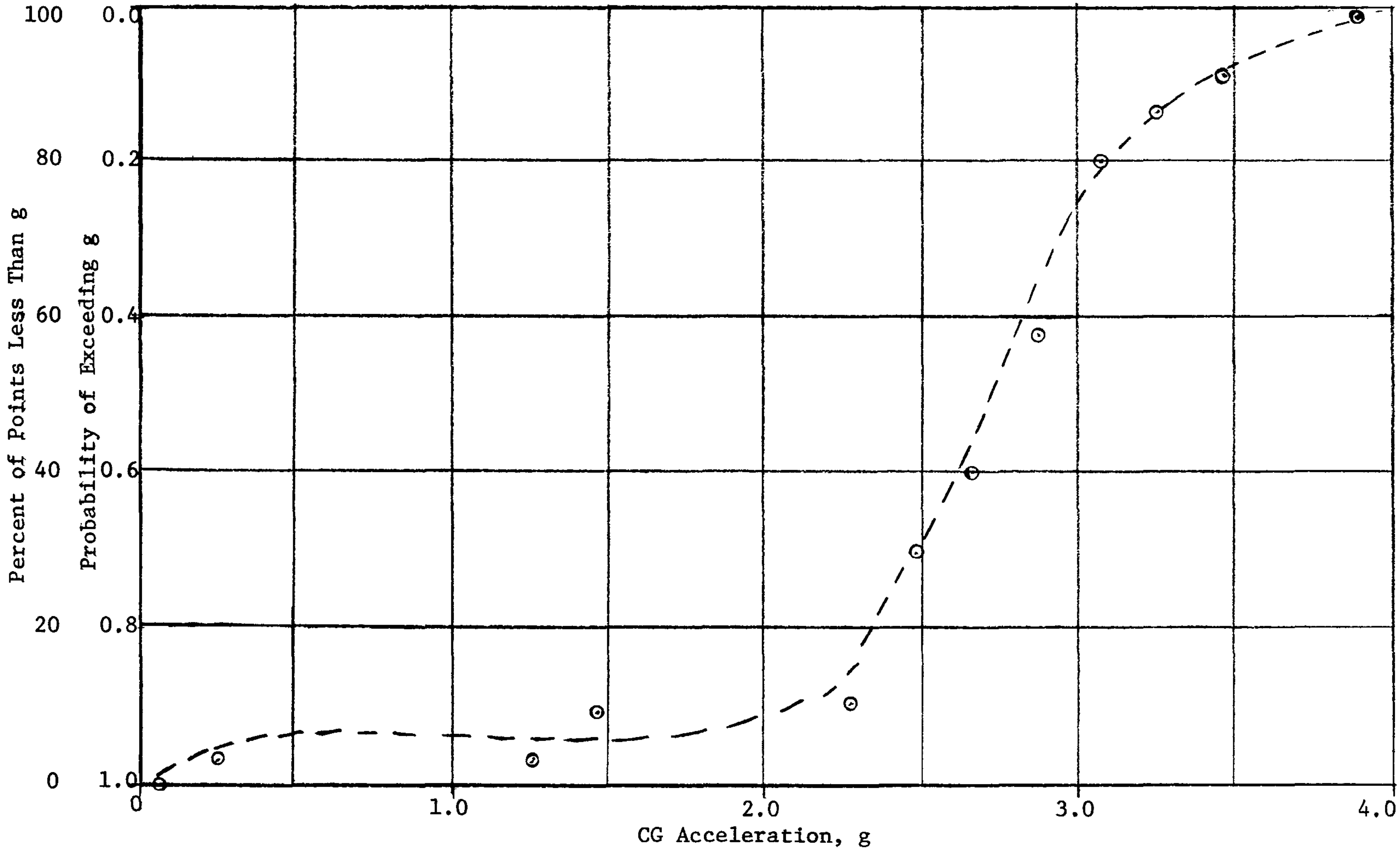


Figure 22. Amplitude Probability Distribution Function for CG Accelerations Incurred at 7.5 mph Over Terrain Profile with RMS Elevation = 5

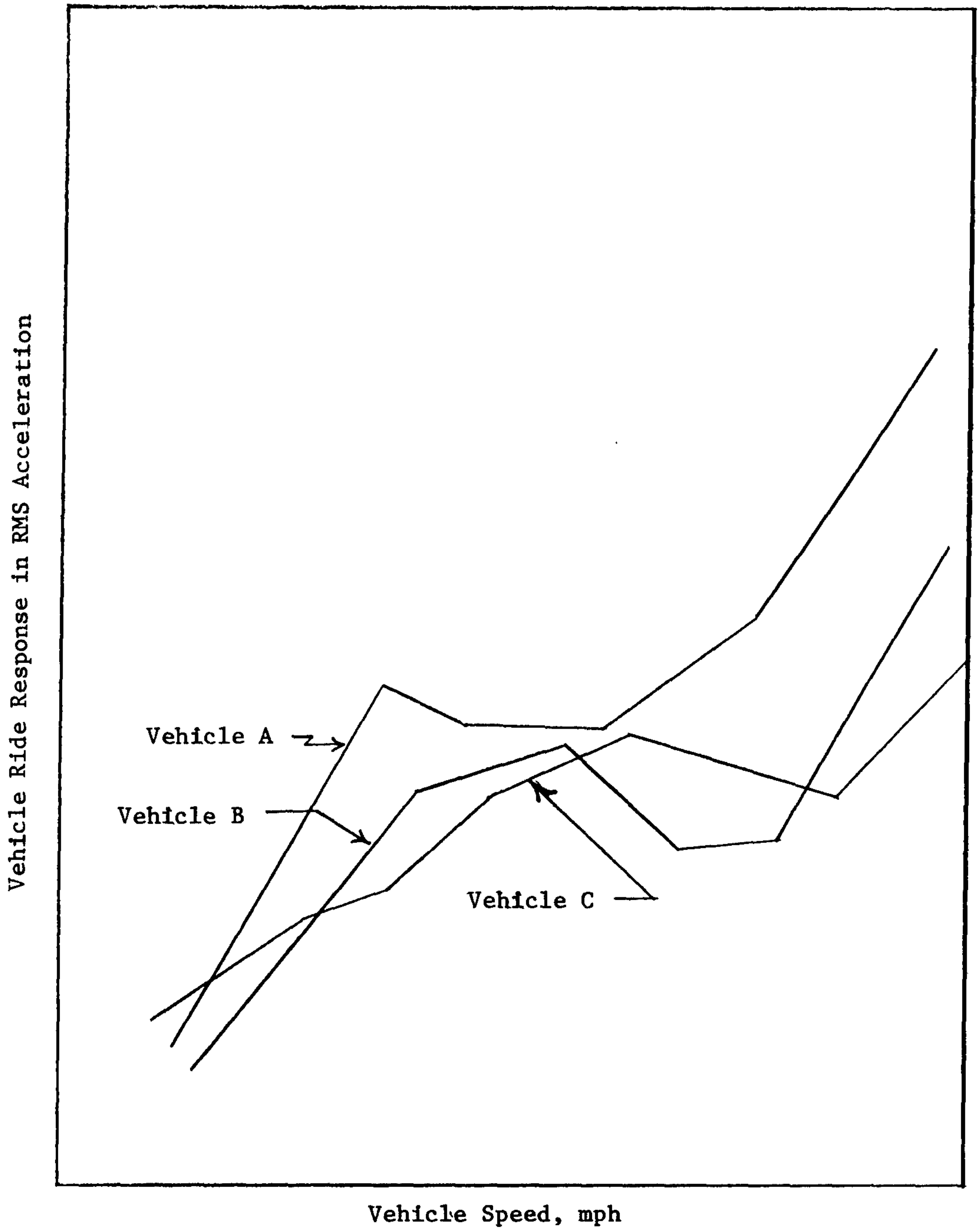


Figure 23. Comparison of Dynamic Response of Three Vehicles Operating in a Given Terrain Environment

response-speed relations for three vehicles operating in a given off-road environment. As before, the complete evaluation as to which vehicle was best suited to the environment would require the APD for each vehicle.

Also, the capability for constructing artificial profiles to conform to any desired power spectrum will permit vehicle response evaluations for remote areas without physical testing of the vehicle in that area. This analysis capability greatly enhances the utility of a comprehensive analytical model for describing the overall mobility aspects of off-road vehicles. It also can serve as an aid in the design of off-road vehicles by permitting the study of simulated motions of candidate vehicles operating on generalized terrain types. The effects of suspension components, for example, can be studied in terms of their vibrational isolation capabilities by examining the ratios of the RMS accelerations of the axles to those at various parts of the vehicle's main frame, or in regard to various independent suspension types by studying the ratios of the RMS fore and aft accelerations.

Application to Real-World Situations

This has been a computer development of the terrain-vehicle-speed relations for a computer model that is representative of an actual vehicle in the military's Table of Organization and Equipment (TO&E) traversing computer-constructed terrain profiles whose power spectra were representative of those of actual profiles. However, the application of this scheme of analysis to actual vehicles traversing actual terrains is necessary to establish its success and practicality. The

main problems that might occur will probably be associated with the terrain input. The terrain profiles in this study were constructed to fulfill certain essential requirements of randomness and stationarity. Tests need to be conducted to establish the degree of stationarity exhibited by representative terrains. A permissible degree of terrain stationarity, which has little effect on the TVS relation, needs to be established. This may turn out to be a uniform standard applicable to all vehicles, or may need to be specified according to vehicle classes, such as wheeled, tracked, etc.

The effect on both the input and output statistics, as well as the frequency distribution, of detrending (removing the longer terrain wavelengths that do not affect ride) the original profile to achieve this adequate degree of stationarity needs to be thoroughly studied. These results should establish the requirements necessary to develop actual TVS relations.

CHAPTER IV: CONCLUSIONS AND RECOMMENDATIONS

Conclusions

On the basis of the results of this study, it is concluded that:

a. Vehicle response statistics obtained from traversing terrains with constant statistics and a given terrain power spectra will be similar.

b. Three basic graphs can be constructed that will completely define the ride dynamics characteristics for a vehicle operating in both on-road and off-road environments.

c. This graphic scheme will rate vehicles, on a standard competitive basis of ride, as to their capability to negotiate cross-country terrains at various speeds.

d. The capability for constructing artificial profiles to conform to any desired power spectrum will (1) permit vehicle response evaluations for remote areas through simulation methods, (2) enhance the utility of a comprehensive analytical model for predicting overall mobility aspects of vehicles, and (3) aid in the design of off-road vehicles.

Recommendations

It is recommended that:

a. The results of this study be applied to describe actual TVS relations.

b. Tests with actual vehicles in real terrain environments

be conducted to determine:

(1) The degree of stationarity exhibited by representative terrains.

(2) The effect of detrending the original terrain to achieve adequate stationarity.

(3) Minimum length of terrain required to insure adequate stationarity of the output response.

(4) The applicability of such TVS evaluations to comprehensive analytical cross-country mobility models.

CHAPTER V: ABSTRACT

Newell Rogers Murphy, Jr., Master of Science, 1971

Major: Engineering Mechanics, College of Engineering

Title of Thesis: A Statistical Analysis of Terrain-Vehicle-Speed Systems

Directed by: Professor Walter R. Carnes

Pages in Thesis, 101 Words in Abstract, 321

Abstract

The specific objective of this study was to define and formulate the procedures for implementing a practical, compact description of terrain-vehicle-speed (TVS) systems and thus establish a methodology for relating measures of ride to measures of terrain roughness and cross-country speed. This methodology should enable the determination of reliable estimates of the average speed a given vehicle can attain under given vibrational constraints as well as the probabilities of exceeding certain specified levels.

This study was entirely a computerized effort. Digital computer simulations were conducted in which a vehicle was run at several selected speeds over terrain profiles with various levels of roughness. The terrain profiles were generated by computer programs that constructed and shaped sequences of random normal numbers to provide representative profiles with a desired power spectrum, variance, mean

value, standard deviation, and RMS. These programs also performed the necessary statistical checks for normality, stationarity, and randomness.

A comprehensive, nonlinear mathematical model of an M37 truck served as the vehicle. This model allowed for the nonlinearities inherent in large rotational motions, the suspension elements, bump stops, loss of ground contact, and the tire compliance, which was represented by a cluster of radially projecting springs.

The results are presented in the form of three basic graphs comparing input and output statistics and distribution functions. The input statistics consisted of terrain roughness as measured by RMS elevation. The output is represented as the RMS vertical acceleration of the vehicle's center of gravity. A family of curves was developed relating vehicle response to terrain roughness for each speed of vehicle traversal. Cross plots of these established relations provided a more useful relation between vehicle response and speed for various degrees of terrain roughness.

The use of such relations to catalog vehicles in terms of their ride dynamics capabilities is suggested. The TVS scheme is readily adaptable to computer models for evaluating and rating the cross-country mobility characteristics of vehicles traversing large, nonhomogeneous terrain areas.

CHAPTER VI: APPENDIX

APPENDIX A: COMPUTER PROGRAMS

The study discussed in this report consisted entirely of a computer effort. The digital computer programs employed in this study are listed on the following pages in the proper sequence for developing the relations established in this thesis. The programs are written in FORTRAN IV programming language for use on a GE-430 computer system in the time sharing mode. A brief description of each program is given in the following paragraphs.

NOISE

The first program listed, NOISE, was developed specifically to compute the random terrain profiles with certain specified statistics. It first calls for a name for the output file of terrain profile points. The input variables are TAU, RMS, ALPHA, FACT1, IX, and III. The variables TAU and ALPHA control the PSD shaping and it was found that setting the ALPHA-TAU product = 0.055 gave the best normalities condition based on results of the Chi-square test. The variable RMS is used to determine the standard deviation of the sequence of profile points (see line No. 1180 in program listing). FACT1 is used to control the RMS roughness level of the resulting profile points. For example, a value of 0.1168 for FACT1 produced a sequence with an RMS = 1.0. To obtain a profile sequence with an RMS roughness of 4.0 requires a value for FACT1 which is (4)(0.1168). The input variable IX is the arbitrary starting point in the random number chain; III is the number of desired profile points.

The NOISE program utilizes two library subroutines: GAUSS and

RANDU. The subroutine RANDU generates a sequence of uniform, random numbers by the conventional congruence method. The subroutine GAUSS generates a sequence of normal, random numbers from the uniform sequence by a technique referred to as the "sum of uniform deviates." A "shift" routine adjusts the numbers to provide an absolute zero mean. The normal, random sequence is passed through a numerical, low-pass filter to yield the final, proper frequency weighted sequence.

STANOR

The next program, STANOR, provides all the necessary tests of the fundamental assumptions. The input to this program is the output file of profile points generated by the NOISE program.

This program first performs the trend test and run test to check the stationarity of the points at the 0.05 level of significance. The sequence of numbers is divided into N equal intervals for the two tests and the mean value computed for each interval. The resulting sequence of the means is checked for the appropriate number of zero crossings (run test) and reverse arrangements (trend test). The Chi-square test is then conducted to check the normality condition. This is also accomplished at the 0.05 level of significance. For this test, the profile sequence is again divided into an appropriate number of groups determined by a formula that depends on the total number of points in the sequence. Three statistical tables, TABK1, TABK2, and TABK3, are used in conjunction with this program.

PSD

The program, PSD, first computes the autocorrelation function from which it obtains the PSD estimate via the Fourier transform of the

autocorrelation function. The input file is the file of numbers of which the PSD is to be obtained. Output files of the correlation coefficient versus lag and PSD estimates versus frequency are generated. The basic input variables are: KKK, LAG, DELTAT, and FACT, where KKK is the number of points in the input file, LAG is the maximum lag (199 for this study) used in computing the autocorrelation function. DELTAT is a value that determines the spacing between the input points and may represent either time or distance. FACT is a coefficient that multiplies each input number (see line No. 1064 in listing). This serves as a means of altering the input numbers by a factor. If no altering is desired, set the value of FACT to unity.

The frequency ω is determined by the formula

$$\omega = \frac{h\pi}{m\Delta t} \qquad h = 0,1,2,. . . m$$

where

m = maximum lags used not to exceed 199

Δt = the equal intervals between points (may be time or space units)

This frequency ω is in units of radians/unit time or distance and must be divided by 2π to obtain frequency in cycles/unit time or distance.

For example, calculate the maximum obtainable spacial frequency for $m = 199$ and $\Delta t = 1$ ft. The maximum frequency is obtained when $h = m = 199$, i.e.

$$\omega_{max} = \frac{\pi}{\Delta t} = \frac{3.14}{1} = 3.14 \text{ rad/ft}$$

or in terms of cycle/ft

$$\Omega_{max} = \frac{3.14}{2\pi} = \frac{3.14}{6.28} = 0.5 \text{ cyc/ft}$$

It is worthy to mention that the raw spectral estimates are smoothed with Hamming coefficients.

TRUCK

The TRUCK program, which represents a two-dimensional mathematical model for the M37, 3/4-ton truck, is a special purpose program written specifically for this study. The program is composed basically of an executive program and two subprograms; DIFFEQ, which contains the Runge-Kutta-Gill solution to the differential equations and ALGEBR, which performs all the required algebraic operations.

The input variables required by the program are clearly defined in the program listing. However, some noteworthy comments concerning the segmented wheel implementation are given to enhance an understanding of the method used to simulate the tire-terrain compliance.

Each wheel, which represented a 9.00x16, 8-PR tire at 45-psi inflation pressure, was divided into ten segments; five on each side of the tire's centerline as shown in Fig. A1. The measured load-deflection relation (Fig. A2) for the 9.00x16 tire at 45-psi inflation pressure was such that a centerline deflection of 1.5 in. required a load of 2860 lb. At this deflection, four spring segments are influenced, two on each side of the centerline (Fig. 1). The spring constant K can be determined from the statics equation:

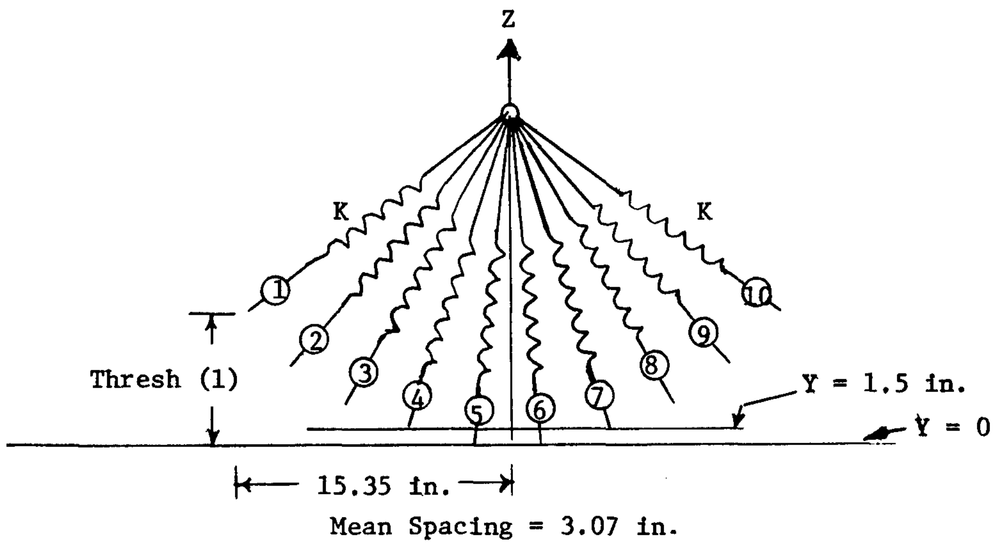


Figure A1. Schematic of Segmented Wheel

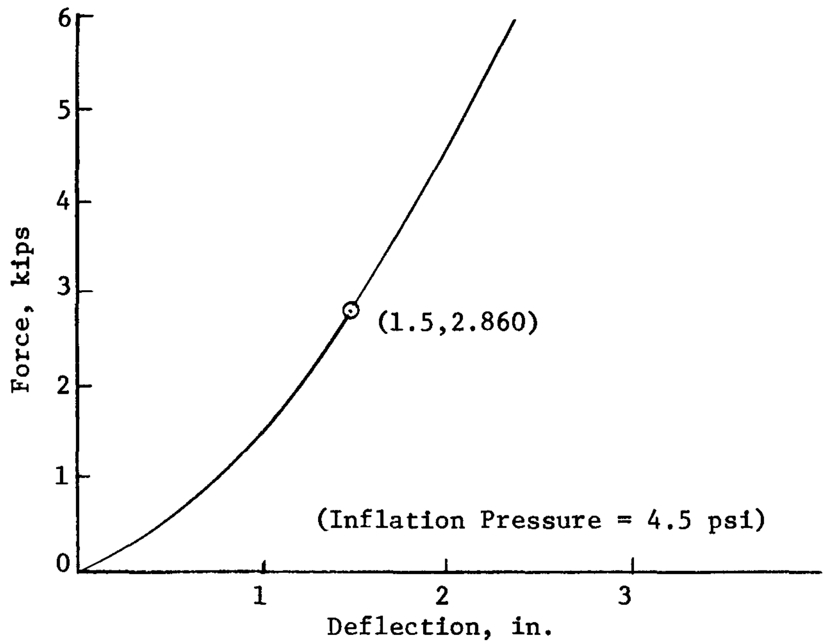


Figure A2. Force Versus Deflection for the 9.00-16, 8-PR Tire

$$F = \sum_{i=1}^{10} K \cos \phi_i \, \Delta_i$$

where

$$\Delta_i = Y_i - \text{THRESH}(i) - Z \, , \qquad Y_i - \text{THRESH}(i) - Z \geq 0$$

$$= 0 \qquad\qquad , \qquad Y_i - \text{THRESH}(i) - Z < 0$$

Y_i = vertical height of terrain profile beneath i^{th} segment

Z = vertical displacement of axle

THRESH(i) = height from the zero reference to the i^{th} spring of the undeflected wheel (see Fig. 1).

For this case and due to the symmetry of the segments about the centerline the equation reduces to

$$2860 = 2K \sum_{i=1}^{2} 1.4 \cos \frac{11.7°}{2} + 0.7 \cos 11.7° + \frac{11.7°}{2}$$

where the effective radial deflections are:

$\Delta_5 = \Delta_6 = 1.4$ in.

$\Delta_4 = \Delta_7 = 0.7$ in.

Solving for K yields

K = 675 lb/in.

Defining GAMMA = K cos ϕ_i = 675 cos ϕ_i yields the following relations for the segments of the front and back wheels:

GAMMA(1) = GAMMA(10) = GAMMA(11) = GAMMA(20) = 411.75

GAMMA(2) = GAMMA(9) = GAMMA(12) = GAMMA(19) = 506.25

GAMMA(3) = GAMMA(8) = GAMMA(13) = GAMMA(18) = 587.25

GAMMA(4) = GAMMA(7) = GAMMA(14) = GAMMA(17) = 648.00

GAMMA(5) = GAMMA(6) = GAMMA(15) = GAMMA(16) = 668.25

A similar relation is derived for the threshold heights of each segment THRESH(i) and these relations are defined in the program.

A mean spacing of 3.07 in. was determined to be adequate for portraying the projected spacing of the springs. As a result, all profile points were spaced 3.07 in. apart and no interpolation scheme was employed to estimate elevation between adjacent points.

The truck was then advanced point by point along the profile and at each advance the resulting motions were computed.

NOISE

```
900$NDM
1000$LIB,GAUSS,,,***
1010C     ****************************************************
1020C     * LOW-PASS GAUSSIAN RANDOM NUMBER GENERATOR        *
1030C     ****************************************************
1110      DIMENSION X(1000),Y(1000)
1130C     ++++++++++++++++++++++++++++++++++++++++++++++++++++
1132      PRINT,"OUTPUT FILE NAME"
1134      READ 108,XNAME
1136 108 FORMAT(A6)
1138      PRINT,"TAU,RMS,ALPHA,FACT1,IX,III"
1140      READ,TAU,RMS,ALPHA,FACT1,IX,III
1180      SIGMAN=RMS*SQRT(1.-EXP(-2.*ALPHA*TAU))
1190      PRINT 102,SIGMAN
1200 102 FORMAT(8HSIGMAN= E16.6)
1210C     ++++++++++++++++++++++++++++++++++++++++++++++++++++
1220C     GENERATE THE RANDOM NORMAL NUMBERS HAVING STANDARD
1230C     DEVIATION SIGMAN.
1235C     *******************
1249C     *******************
1250      AM=0.
1260      DO 200 I=1,III
1262      SIGSQ=SIGMAN*SIGMAN
1270      CALL GAUSS(IX,SIGSQ,AM,V)
1280 200 X(I)=V
1281      CALL SHIFT(X,III,XBAR)
1282      PRINT 210,XBAR
1283 210 FORMAT("XBAR AFTER GAUSS="E16.6)
1285      CALL STDEV(X,III,DEV)
1286      RMSRAD=SQRT(DEV)*3.1415926536*2.
1287      PRINT 220,DEV,RMSRAD
1289 220 FORMAT("DEV="E16.6,"RMSRAD="E16.6)
1290      PRINT 106
1300 106 FORMAT("RANDOM NORMAL NUMBERS WITH STAND. DEV. SIGMAN")
1320 103 FORMAT(E20.10)
1332C     ++++++++++++++++++++++++++++++++++++++++++++++++++
1334C     STANDARD DEVIATION IS COMPUTED TO SEE WHETHER
1336C      SUBROUTINE GAUSS(IX,SIGMAN,AM,V) GENERATES CORRECT
1338C     RESULTS.
1340      CALL STDEV(X,III,DEV)
1342      PRINT 330,DEV
1344 330 FORMAT("COMPUTED STANDARD DEVIATION= " E16.8)
1350C     STANDARD DEV. BLOCK IS TO BE INSERTED.
1360C     ++++++++++++++++++++++++++++++++++++++++++++++++++++
1370C     GENERATE THE LOW-PASS GAUSSIAN
1375      CALL SHIFT(X,III,XBAR)
1379C     ***************************
1380      AA=EXP(-ALPHA*TAU)
1381C     ***************************
1390      Y(1)=0.0
```

NOISE CONTINUED

```
      DO 300 I=2,III
  300 Y(I)=X(I)+Y(I-1)*AA
      DO 401 I=1,III
  401 Y(I)=FACT1*Y(I)
      WRITE(2,103)(Y(I),I=1,III)
      CALL CLOSEF(2,XNAME)
      PRINT 107
  107 FORMAT("LOW-PASS GAUSSIAN IS GENERATED AND STORED")
      END
C     ++++++++++++++++++++++++++++++++++++++++++++++++++++
      SUBROUTINE RANDU(IX,IY,YFL)
      IY=IX*4099
      IF(IY)5,6,6
    5 IY=IY+8388607+1
    6 YFL=IY
      YFL=YFL*1.192093E-7
      RETURN
      END
C     ++++++++++++++++++++++++++++++++++++++++++++++++++++
      SUBROUTINE STDEV(X,III,DEV)
      DIMENSION X(1)
      SUM=0.0
      DO 420 I=1,III
  420 SUM=SUM+X(I)
      SUM1=SUM*SUM
      SUM2=0.0
      DO 440 I=1,III
  440 SUM2=SUM2+X(I)*X(I)
      AIII=III
      SD=SQRT((SUM2-SUM1/AIII)/(AIII-1.))
      DEV=SD
      RETURN
      END
C     ++++++++++++++++++++++++++++++++++++++++++++++++++++
      SUBROUTINE SHIFT(X,III,XBAR)
      DIMENSION X(1)
      SUM=0.0
      DO 300 I=1,III
      SUM=SUM+X(I)
  300 CONTINUE
      AIII=III
      XBAR=SUM/AIII
      PRINT 310,XBAR
  310 FORMAT("THE TIME SERIES IS SHIFTED BY XBAR="E16.8)
      DO 320 I=1,III
      X(I)=X(I)-XBAR
  320 CONTINUE
      RETURN
      END
```

GAUSS

```
C
C.................................................................
C
C SUBROUTINE GAUSS
C
C PURPOSE
C    COMPUTES A NORMALLY DISTRIBUTED RANDOM NUMBER WITH A GIVEN
C    MEAN AND STANDARD DEVIATION
C
C USAGE
C    CALL GAUSS(IX,S,AM,V)
C
C DESCRIPTION OF PARAMETERS
C    IX -IX MUST CONTAIN AN ODD INTEGER NUMBER WITH NINE OR
C        LESS DIGITS ON THE FIRST ENTRY TO GAUSS. THEREAFTER
C        IT WILL CONTAIN A UNIFORMLY DISTRIBUTED INTEGER RANDOM
C        NUMBER GENERATED BY THE SUBROUTINE FOR USE ON THE NEXT
C        ENTRY TO THE SUBROUTINE.
C    S  -THE DESIRED STANDARD DEVIATION OF THE NORMAL
C        DISTRIBUTION.
C    AM -THE DESIRED MEAN OF THE NORMAL DISTRIBUTION
C    V  -THE VALUE OF THE COMPUTED NORMAL RANDOM VARIABLE
C
C REMARKS
C    THIS SUBROUTINE USES RANDU WHICH IS MACHINE SPECIFIC
C
C SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C    RANDU
C
C METHOD
C    USES 12 UNIFORM RANDOM NUMBERS TO COMPUTE NORMAL RANDOM
C    NUMBERS BY CENTRAL LIMIT THEOREM. THE RESULT IS THEN
C    ADJUSTED TO MATCH THE GIVEN MEAN AND STANDARD DEVIATION.
C    THE UNIFORM RANDOM NUMBERS COMPUTED WITHIN THE SUBROUTINE
C    ARE FOUND BY THE POWER RESIDUE METHOD.
C
C.................................................................
C
SUBROUTINEGAUSS(IX,S,AM,V)
A=0.0
DO50I=1,12
CALLRANDU(IX,IY,Y)
IX=IY
50A=A+Y
V=(A-6.0)*S+AM
RETURN
END
```

RANDU

```
C
C...............................................................
C
C SUBROUTINE RANDU
C
C PURPOSE
C    COMPUTES UNIFORMLY DISTRIBUTED RANDOM REAL NUMBERS BETWEEN
C    0 AND 1.0 AND RANDOM INTEGERS BETWEEN ZERO AND
C    2**31. EACH ENTRY USES AS INPUT AN INTEGER RANDOM NUMBER
C    AND PRODUCES A NEW INTEGER AND REAL RANDOM NUMBER.
C
C USAGE
C    CALL RANDU(IX,IY,YFL)
C
C DESCRIPTION OF PARAMETERS
C    IX - FOR THE FIRST ENTRY THIS MUST CONTAIN ANY ODD INTEGER
C         NUMBER WITH NINE OR LESS DIGITS. AFTER THE FIRST ENTRY,
C         IX SHOULD BE THE PREVIOUS VALUE OF IY COMPUTED BY THIS
C         SUBROUTINE.
C    IY - A RESULTANT INTEGER RANDOM NUMBER REQUIRED FOR THE NEXT
C         ENTRY TO THIS SUBROUTINE. THE RANGE OF THIS NUMBER IS
C         BETWEEN ZERO AND 2**31
C    YFL- THE RESULTANT UNIFORMLY DISTRIBUTED, FLOATING POINT,
C         RANDOM NUMBER IN THE RANGE 0 TO 1.0
C
C REMARKS
C    THIS SUBROUTINE IS SPECIFIC TO THE GE-427
C    THE SUBROUTINE SHOULD PRODUCE (2**22)-1 TERMS
C    BEFORE REPEATING
C
C SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C    NONE
C
C METHOD
C    POWER RESIDUE METHOD DISCUSSED IN IBM MANUAL C20-8011,
C    RANDOM NUMBER GENERATION AND TESTING
C
C...............................................................
C
 SUBROUTINERANDU(IX,IY,YFL)
 IY=IX*4099
 IF(IY)5,6,6
 5IY=IY+8388607+1
 6YFL=IY
 YFL=YFL*1.192093E-7
 RETURN
 END
```

STANOR

```
120$NDM
130C     ************************************
140C     * STATIONARITY AND NORMALITY TEST *
150C     ************************************
160 DIMENSIONX(1000)
165 DIMENSIONZ(50),T(50)
170 DIMENSIONTF(50)
180 DIMENSIONTABLE1(31,7),TABLE2(15,7),TABLE3(2,7)
185 COMMONMTEST
186 MTEST=0
190 100FORMAT(8I10)
195 109FORMAT(7F9.3)
200 101FORMAT(8F10.3)
210 102FORMAT(8E12.4)
220 103FORMAT(4H IP=I5,8E12.4)
230 104FORMAT(8F10.5)
240C      TABLE1(I,J)---PERCENTAGE POINTS OF RUN DISTRIBUTION
250C      TABLE3(I,J)---GROUP NUMBER DECISION TABLE
260C      TABLE2(I,J)---PERCENTAGE POINTS OF REVERSE ARRANGEMENT DISTRIBU
270C      READ THE TIME SERIES
271 PRINT,"FILE NAME"
272 READ839,XDNAME
273 839FORMAT(A6)
275 PRINT,"NO. POINTS, FACTOR"
280 READ,III
290 AIII=III
300 IP=1000
310 PRINT745,III
311 745FORMAT(5H III=I10)
315 CALLOPENF(1,XDNAME)
318 999FORMAT(E20.10)
320 READ(1,999)(X(I),I=1,III)
321C      FACTOR IS MULTIPLIED FOR THE DATA X444
360C      * * * * * * * * * * * * * * * * * * * * * * * *
370C      READ THE PERCENTAGE POINTS OF RUN TEST TABLE
380 CALLOPENF(2,"TABKA1")
390 READ(2,)((TABLE1(K1,K2),K2=1,7),K1=1,31)
440C      * * * * * * * * * * * * * * * * * * * *
450C      READ THE PERCENTAGE POINTS OF REVERSE ARRANGE DISTRIBUTION TABLE
460 CALLOPENF(3,"TABKA2")
470 READ(3,)((TABLE2(K1,K2),K2=1,7),K1=1,15)
520C      * * * * * * * * * * * * * * * * * * * *
530C      READ THE GROUP NUMBER DECISION TABLE
540 CALLOPENF(4,"TABKA3")
550 READ(4,)((TABLE3(K1,K2),K2=1,7),K1=1,2)
600C      NUMBER = NUMBER IN SUBSET
610 NUMBER=1
620C      * * * * * * * * * * * * * * * * * * * * *
630 600CONTINUE
640C      DETERMINE NUMBER OF GROUP NG
```

STANOR CONTINUED

```
650 IF(200-III)339,339,340
660 340ANG=AIII*(16.0/200.)
670 IP=3001
680 GOTO390
690 339IF(III-2000)349,350,350
700 350ANG=1.87*(AIII-1.)**(2.0/5.0)
710 IP=3002
720 GOTO390
730 349CONTINUE
740 K2=1
750 360CONTINUE
760 IF(AIII-TABLE3(1,K2))361,362,363
770 361ANG=((TABLE3(2,K2+1)-TABLE3(2,K2))/(TABLE3(1,K2+1)-TABLE3(1,K2))
780&)
790&*(AIII-TABLE3(1,K2))+TABLE3(2,K2)
800 IP=3003
810 GOTO390
820 362ANG=TABLE3(2,K2)
830 IP=3004
840 GOTO390
850 363K2=K2+1
860 GOTO360
870 390NG=ANG
880 PRINT801,NG
885 801FORMAT(17H NUMBER OF GROUP=I5)
890 CALLGROUP(X,III,ZSTEP,NG,XMIN,XMAX,Z)
900 CALLAGROUP(X,III,T,NG,XXMIN,XXMAX,ASTEP)
910 CALLTRTEST(T,NG,XXMIN,XXMAX,TABLE2,IPASS)
920 CALLRNTEST(T,NG,XXMIN,XXMAX,TABLE1,JPASS)
930C     * * * * *  TEST FOR NORMALITY * * * * * * * * * * * * * * * * * **
940 CALLCHITN(X,III,NG,XMIN,XMAX,ZSTEP,Z,TF,XBAR,S,CHI2,PCHI)
950 PRINT401,PCHI
960 401FORMAT(19H CHI-SQUARE VALUE =E12.4)
970 IF(PCHI-0.05)409,410,410
980 410PRINT420
990 420FORMAT(30H NORMALITY TEST PASSED        )
1000 GOTO499
1010 409PRINT430
1020 430FORMAT(30H NORMALITY TEST NOT PASSED    )
1240 499CONTINUE
1250 END
1260 SUBROUTINEGROUP(X,III,ZSTEP,NG,XXMIN,XXMAX,Z)
1265 COMMONMTEST
1270 DIMENSIONX(2000),Z(50)
1280 104FORMAT(I5,4E12.4)
1290 ANG=NG
1300 AIII=III
1310 XXMAX=-10.E10
1320 XXMIN=10.E10
1330 DO200I=1,III
```

STANOR CONTINUED

```
TEMP1=AMAX1(XXMAX,X(I))
XXMAX=TEMP1
TEMP2=AMIN1(XXMIN,X(I))
200XXMIN=TEMP2
IP=4000
XXMAX=XXMAX+10.E-10
XXMIN=XXMIN-10.E-10
ZSTEP=(XXMAX-XXMIN)/ANG
TEMP1=XXMIN
TEMP2=TEMP1+ZSTEP
IF(MTEST.NE.0)GOTO444
107FORMAT(/,6H GROUP)
444CONTINUE
DO300N=1,NG
Z(N)=0.0
DO310I=1,III
IF(X(I)-TEMP1)310,310,311
311IF(TEMP2-X(I))310,310,315
315Z(N)=Z(N)+1.0
310CONTINUE
IP=4008
AN=N
IF(MTEST.NE.0)GOTO445
PRINT104,N,TEMP1,TEMP2,Z(N)
445CONTINUE
TEMP1=TEMP2
300TEMP2=TEMP2+ZSTEP
MTEST=MTEST+1
RETURN
END
SUBROUTINETRTEST(Z,NG,XXMIN,XXMAX,TABLE2,IPASS)
DIMENSIONZ(50),A(50),TABLE2(15,7)
104FORMAT(4H IP=I5,8E12.4)
108FORMAT(I5,F10.0)
C      * * * * * * * * * * * * * *REVERSE ARRANGEMENT COMPUTATION
PRINT109
109FORMAT(/,21H REVERSE ARRANGEMENT )
NGM1=NG-1
ANG=NG
DO200N=1,NGM1
A(N)=0.0
NP1=N+1
DO210J=NP1,NG
IF(Z(J)-Z(N))220,210,210
220A(N)=A(N)+1.0
210CONTINUE
PRINT108,N,A(N)
200CONTINUE
C      * * * * * * * TOTAL OF REVERSE ARRANGEMENT
TREV=0.0
```

STANOR CONTINUED

```
1770 DO250N=1,NGM1
1780 250TREV=TREV+A(N)
1790 PRINT773
1792 773FORMAT(/,"SUMMARY OF TREND TEST")
1794 PRINT774,TREV
1796 774FORMAT(" TOTAL REVERSE ARRANGEMENT = "E12.4)
1810C      NOTE THAT IT IS TEMPORARY ASSUMEDE THAT K2L=3,AND K2U=5 IN THE
1820C      TABLE2(I,J) ARRAY
1830C      * * *COMPUTE THE LOWER LIMIT BY LINEAR INTERPOLATION. (ASSUME [illegible]
 1840C            IS GREATER THAN 10 AND LESS THAN 100)
1850 IF(NG-10)309,309,301
1860 301IF(100-NG)309,309,302
1870 309PRINT308,NG
1880 308FORMAT(48H NG IS GREATER THAN 100 OR LESS THAN 10.    NG =I10)
 1890 STOP
1900 302CONTINUE
1910 DO641K1=2,15
1915 IF(ANG.GT.TABLE2(K1,1))GOTO641
1920 IF(ANG.EQ.TABLE2(K1,1))GOTO642
1925 XX1=TABLE2(K1-1,1)
1930 XX2=TABLE2(K1,1)
1935 Y1=TABLE2(K1-1,3)
1936 Y2=TABLE2(K1,3)
1940 ALOW=(Y2-Y1)*(ANG-XX1)/(XX2-XX1)+Y1
1945 YY1=TABLE2(K1-1,6)
1946 YY2=TABLE2(K1,6)
1950 AUP=(YY2-YY1)*(ANG-XX1)/(XX2-XX1)+YY1
1953 GOTO643
1955 642ALOW=TABLE2(K1,3)
1960 AUP=TABLE2(K1,6)
1965 GOTO643
1970 641CONTINUE
1975 643CONTINUE
2034 PRINT775,ALOW
2036 775FORMAT(" LOWER LIMIT OF REVERSE ARRANGEMENT = "E12.4)
2038 PRINT776,AUP
2040 776FORMAT(" UPPER LIMIT OF REVERSE ARRANGEMENT = "E12.4)
2045 510FORMAT(6H ALOW=E12.4,5H AUP=E12.4,6H TREV=E12.4)
2050C      TEST WHETHER ANG IS BETWEEN THE LIMIT
2060 IF(TREV-ALOW)390,400,400
2070 400IF(AUP-TREV)390,410,410
2080 390IPASS=0
2090 PRINT391
2100 391FORMAT(" TREND TEST IS NOT PASSED")
2110 GOTO420
2120 410IPASS=1
2130 PRINT411
2140 411FORMAT(" TREND TEST IS PASSED ")
2150 420CONTINUE
2160 RETURN
```

STANOR CONTINUED

```
2170 END
2180 SUBROUTINEAGROUP(X,III,T,NG,XXMIN,XXMAX,ASTEP)
2190C      THE PURPOSE OF AGROUP IS TO GROUP THE SERIES X(I) AND AVERAGE I
2200C      AND STOR THEM IN T(I) ARRAY. NOTICE THE DIFFERENCE BETWEEN GROB
2210C      AND AGROUP.
2220 DIMENSIONX(2000),T(50)
2230 104FORMAT(4H IP=I10,8E12.4)
2240 AIII=III
2250 ANG=NG
2260C      COMPUTE THE INTERVAL IN FLOATING MODE
2270 TEMP1=AIII/ANG
2280 ASTEP=TEMP1
2290C      THESE CODES MAY BE TAKEN OUT * * * * *
2350C
2360 152FORMAT(7F8.2)
2370C      * * * * * * * * * * * * * * * * * * *
2380 TEMP2=XXMIN
2390 DO200N=1,NG
2400 T(N)=0.0
2410 AN=N
2420 NT=0
2430 TEMP3=TEMP1*(AN-1.0)
2440 TEMP4=TEMP1*AN+1.0E-10
2450 DO210I=1,III
2460 AI=I
2470 IF(AI-TEMP3)219,219,220
2480 220IF(TEMP4-AI)219,219,230
2490 230T(N)=T(N)+X(I)
2500 NT=NT+1
2510 219CONTINUE
2520 210CONTINUE
2530C      * * * * * * * PRINT THE INTERMEDIATE STEP
2560C      * * * * * * *
2570 ANT=NT
2580 T(N)=T(N)/ANT
2610 200CONTINUE
2615 PRINT310
2616 310FORMAT(/,23H AVERAGED TIME HISTORY )
2617 DO311N=1,NG
2618 311PRINT312,N,T(N)
2619 312FORMAT(I5,E12.4)
2620 RETURN
2630 END
2640 SUBROUTINERNTEST(X,NG,XXMIN,XXMAX,TABLE1,JPASS)
2650C      THE MAIN PROGRAM CALL RNTEST(Z,NG,XXMIN,XXMAX,TABLE1,JPASS)
2660 DIMENSIONX(50),TABLE1(31,7),X1(50)
2670 104FORMAT(4H IP=I5,4E12.4)
2680C      *********COMPUTE MEAN VALUE
2683 189CONTINUE
2684 187FORMAT(I5,3E12.4)
```

STANOR CONTINUED

```
TEMP=0.0
DO800N=1,NG
800TEMP=TEMP+X(N)
ANG=NG
XMEAN=TEMP/ANG
C      ********DETERMINE NUMBER OF RUN
NRUN=1
IF(X(1)-XMEAN)267,268,268
267ICHG=-1
GOTO270
268ICHG=1
270CONTINUE
DO260I=2,NG
IF(X(I)-XMEAN)262,261,261
261K=1
GOTO263
262K=-1
GOTO263
263IF(ICHG-K)265,260,265
265NRUN=NRUN+1
ICHG=K
260CONTINUE
C      *******COMPUTE LOWER LIMIT BY LINER INTERPOLATION
C               (ASSUME NG/2 IS GREATER THAN 5 AND LESS THAN 100)
NNG=NG/2
IF(NNG-5)309,309,301
301IF(100-NNG)309,309,302
309PRINT308,NNG
308FORMAT(45H NNG IS GREATER THAN 100 OR LESS THAN 5.  NG=I10)
STOP
302CONTINUE
ANNG=NNG
DO641K1=2,31
IF(ANNG.GT.TABLE1(K1,1))GOTO641
IF(ANNG.EQ.TABLE1(K1,1))GOTO642
XX1=TABLE1(K1-1,1)
XX2=TABLE1(K1,1)
Y1=TABLE1(K1-1,3)
Y2=TABLE1(K1,3)
ALOW=(Y2-Y1)*(ANNG-XX1)/(XX2-XX1)+Y1
YY1=TABLE1(K1-1,6)
YY2=TABLE1(K1,6)
AUP=(YY2-YY1)*(ANNG-XX1)/(XX2-XX1)+YY1
GOTO643
642ALOW=TABLE1(K1,3)
AUP=TABLE1(K1,6)
GOTO643
641CONTINUE
643CONTINUE
PRINT790
```

STANOR CONTINUED

```
3176 790FORMAT(/,"SUMMARY OF RUN TEST")
3178 PRINT791,NRUN
3180 791FORMAT(" TOTAL NUMBER OF RUN = "I10)
3181 PRINT793,AUP
3182 PRINT792,ALOW
3184 792FORMAT(" LOWER LIMIT OF RUN ="E12.4)
3185 510FORMAT(6H ALOW=E12.4,5H AUP=E12.4,6H NRUN=E12.4)
3186 793FORMAT(" UPPER LIMIT OF RUN = "E12.4)
3190C      TEST WHETHER ANNG IS BETWEEN THE LIMITS
3200 IF(ANNG-ALOW)390,400,400
3210 400IF(AUP-ANNG)390,410,410
3220 390IPASS=0
3230 PRINT391
3240 391FORMAT(" RUN TEST IS NOT PASSED ")
3250 GOTO420
3260 410IPASS=1
3270 PRINT411
3280 411FORMAT(" RUN TEST IS PASSED ")
3290 420CONTINUE
3300 RETURN
3310 END
3320 SUBROUTINECHITN(X,N,NG,XMIN,XMAX,STEP,F,TF,XBAR,S,CHI2,PCHI)
3330 DIMENSIONF(1),TF(1),X(1)
3340 2FORMAT(4X10F10.6)
3350 SX=0.
3352 PRINT791
3354 791FORMAT(/,"SUMMARY OF NORMALITY TEST")
3360 SX2=0.
3370 DO200I=1,N
3380 SX=SX+X(I)
3390 SX2=SX2+X(I)*X(I)
3400 200CONTINUE
3410 XBAR=SX/N
3420 S=SQRT((SX2-(SX*SX/N))/(N-1))
3430 CALLGROUP(X,N,STEP,NG,XMIN,XMAX,F)
3440 X1=(XMIN-XBAR)/S
3450 TEMP=STEP/S
3460 DO210I=1,NG
3470 X2=X1+TEMP
3480 A1=RNORM(X1)
3490 A2=RNORM(X2)
3500 TF(I)=(A2-A1)*N
3510 PRINT2,X1,X2,A1,A2,TF(I)
3520 X1=X2
3530 210CONTINUE
3540 CHI2=0.
3550 DO220I=1,NG
3560 CHI2=CHI2+((F(I)-TF(I))**2)/TF(I)
3570 220CONTINUE
3580 DF=NG-1
```

STANOR CONTINUED

```
PCHI=CHIPR(DF,CHI2)
RETURN
END
FUNCTIONRNORM(Z)
DIMENSIONA(6)
A(1)=4.9867347E-2
A(2)=2.11410061E-2
A(3)=3.2776263E-3
A(4)=3.80036E-5
A(5)=4.88906E-5
A(6)=5.383E-6
IF(Z)100,110,120
100CONTINUE
ASSIGN160TOJEL
Z1=-Z
GOTO130
110CONTINUE
B=.5
GOTO160
120CONTINUE
ASSIGN150TOJEL
Z1=Z
130CONTINUE
B=1.
DO140I=1,6
B=B+A(I)*Z1**I
140CONTINUE
B=.5/(B**16)
GOTOJEL,(150,160)
150CONTINUE
B=1.-B
160CONTINUE
RNORM=B
RETURN
END
CCHIPR      CHISQUARE PROBABILITY
FUNCTIONCHIPR(DF,CHSQ)
A=.5*DF
X=.5*CHSQ
IF(X)90,90,100
90CHIPR=1.
GOTO170
100TERM=1.
SUM=0.
COFN=A
IF(13.-X)110,110,120
110IF(A-X)140,140,120
C      CONVERGENT SERIES FOR X .LT. A OR .LT. 13.
120CON=1.
FACT=-A
```

STANOR CONTINUED

```
130TEMP=SUM
SUM=SUM+TERM
COFN=COFN+1
TERM=TERM*X/COFN
IF(SUM-TEMP)160,160,130
C     ASYMPTOTIC SERIES FOR X .GTE. A AND X .GTE. 13.
140CON=0.
FACT=X
150TEMP=SUM
SUM=SUM+TERM
COFN=COFN-1
RATIO=COFN/X
TERM=TERM*RATIO
IF(SUM-TEMP)160,160,150
160CHIPR=CON+EXP(ALOG(SUM)-X+A*ALOG(X)-ALOG(GAMMA(A,0.)))/FACT
170CONTINUE
C     ****************************************
C     ****************************************
RETURN
END
FUNCTIONGAMMA(U,V)
DIMENSIONB(8)
B(1)=.035868343
B(2)=-.193527818
B(3)=.482199394
B(4)=-.756704078
B(5)=.918206857
B(6)=-.897056937
B(7)=.988205891
B(8)=-.577191652
A=U
X=V
XN=15.0
1IF(A)4,2,3
2IF(X-1.0)4000,4000,5000
4Z=AINT(A)
Y=ABS(Z-A)
EPS=3.0E-8
IF(Y-EPS)21,21,5
21A=Z
GOTO22
5Y=1.0-Y
IF(Y-EPS)6,6,3
6A=Z-1.0
22IF(X-1.0)4000,4000,5000
3IF(X)7,1000,7
7IF(X-1.0)9,9,8
8IF(A)5000,10,10
10IF((X/A)-1.0)1000,5000,5000
9IF(ABS(A)-1.)11,1000,1000
```

STANOR CONTINUED

```
11IF(X-.1)1000,12,12
12IF(X-.2)13,14,14
13XN=140.0
GOTO5000
14IF(X-.4)15,16,16
15XN=80.0
GOTO5000
16IF(X-.6)17,18,18
17XN=60.0
GOTO5000
18IF(X-.8)19,20,20
19XN=40.0
GOTO5000
20XN=20.0
5000XFT=X
N=XN
DO5001I=1,N
XFT=(XN/XFT)+1.0
XFT=((XN-A)/XFT)+X
5001XN=XN-1.0
TEM=(ALOG(X)*A)-X
XFT=EXP(TEM)/XFT
ANS=XFT
GOTO6000
4000SUM=0.0
EPS=1.0E-8
XM=1.0
XMT=1.0
EX=-1.0
TEM=X
4001Y=TEM/(XMT*XM)
4011SUM=SUM+EX*Y
IF(ABS(Y)-EPS)4003,4002,4002
4002TEM=TEM*X
XM=XM+1.0
XMT=XMT*XM
EX=-EX
GOTO4001
4003E=-ALOG(X)-.57721566-SUM
IF(A)4005,4004,4005
4004ANS=E
GOTO6000
4005IF(A+2.0)4010,4009,4008
4008ANS=-E+EXP(-X)/X
GOTO6000
4009SUM=(1.0/X)-(1.0/(X*X))
ANS=(E-EXP(-X)*SUM)*.5
GOTO6000
4010EX=-1.0
N=-A-1.0
```

STANOR CONTINUED

```
XMT=1.0
SUM=0.0
TEM=X
DO4006I=1,N
TEM=TEM*X
XM=I+1
Y=XMT/TEM
SUM=SUM+EX*Y
EX=-EX
4006XMT=XMT*XM
SUM=SUM+1.0/X
Z=E-EXP(-X)*SUM
Y=ABS(A)
EX=-1.0
XMT=1.0
N=Y
DO4007I=2,N
XM=I
XMT=XMT*XM
4007EX=-EX
ANS=Z*EX/XMT
GOTO6000
1000IF(A-1.0)1001,1003,1004
1004IF(A-2.0)1003,1003,1002
1003AT=A-1.0
ATM=B(1)*AT
DO1005I=2,8
1005ATM=(ATM+B(I))*AT
ANS=ATM+1.0
GOTO2000
1001IF(A)1007,1006,1006
1006BN=A
BS=A
GOTO1010
1007BN=A
BS=A
1011IF(BS)1009,1010,1010
1009BS=BS+1.0
BN=BS*BN
GOTO1011
1010ATM=BS*B(1)
DO1012I=2,8
1012ATM=(ATM+B(I))*BS
ANS=(ATM+1.0)/BN
GOTO2000
1002BN=A-1.0
BS=BN
1013IF(2.0-BS)1015,1014,1014
1015BS=BS-1.0
BN=BS*BN
```

STANOR CONTINUED

```
GOTO1013
1014BS=BS-1.0
ATM=BS*B(1)
DO1016I=2,8
1016ATM=(ATM+B(I))*BS
ANS=(ATM+1.0)*BN
2000IF(X)2001,6000,2001
2001W=1.0
XM=1.0
SUM=1.0
EPS=1.0E-8
2002W=(X/(A+XM))*W
SUM=SUM+W
XM=XM+1.0
IF(ABS(W)-EPS)2003,2002,2002
2003T=A*ALOG(X)
T=EXP(T-X)/A
ANS=ANS-(T*SUM)
6000GAMMA=ANS
RETURN
END
```

TABKA 1

```
100 0.,0.99,0.975,0.95,0.05,0.025,0.01,
110 5.,2.,2.,3.,8.,9.,9.,
120 6.,2.,3.,3.,10.,10.,11.,
130 7.,3.,3.,4.,11.,12.,12.,
140 8.,4.,4.,5.,12.,13.,13.,
150 9.,4.,5.,6.,13.,14.,15.,
160 10.,5.,6.,6.,15.,15.,16.,
170 11.,6.,7.,7.,16.,16.,17.,
180 12.,7.,7.,8.,17.,18.,18.,
190 13.,7.,8.,9.,18.,19.,20.,
200 14.,8.,9.,10.,19.,20.,21.,
210 15.,9.,10.,11.,20.,21.,22.,
220 16.,10.,11.,11.,22.,22.,23.,
230 18.,11.,12.,13.,24.,25.,26.,
240 20.,13.,14.,15.,26.,27.,28.,
250 25.,17.,18.,19.,32.,33.,34.,
260 30.,21.,22.,24.,37.,39.,40.,
270 35.,25.,27.,28.,43.,44.,46.,
280 40.,30.,31.,33.,48.,50.,51.,
290 45.,34.,36.,37.,54.,55.,57.,
300 50.,38.,40.,42.,59.,61.,63.,
310 55.,43.,45.,46.,65.,66.,68.,
320 60.,47.,49.,51.,70.,72.,74.,
330 65.,52.,54.,56.,75.,77.,79.,
340 70.,56.,58.,60.,81.,83.,85.,
350 75.,61.,63.,65.,86.,88.,90.,
360 80.,65.,68.,70.,91.,93.,96.,
370 85.,70.,72.,74.,97.,99.,101.,
380 90.,74.,77.,79.,102.,104.,107.,
390 95.,79.,82.,84.,107.,109.,112.,
400 100.,84.,86.,88.,113.,115.,117.
```

TABKA2

```
100 0.,0.99,0.975,0.95,0.05,0.025,0.01,
110 10.,9.,11.,13.,31.,33.,35.,
120 12.,16.,18.,21.,44.,47.,49.,
130 14.,24.,27.,30.,60.,63.,66.,
140 16.,34.,38.,41.,78.,81.,85.,
150 18.,45.,30.,54.,98.,102.,107.,
160 20.,59.,64.,69.,120.,125.,130.,
170 30.,152.,162.,171.,263.,272.,282.,
180 40.,290.,305.,319.,460.,474.,489.,
190 50.,473.,495.,514.,710.,729.,751.,
200 60.,702.,731.,756.,1013.,1038.,1067.,
210 70.,977.,1014.,1045.,1369.,1400.,1437.,
220 80.,1299.,1344.,1382.,1777.,1815.,1860.,
230 90.,1688.,1721.,1766.,2238.,2283.,2336.,
240 100.,2083.,2145.,2198.,2751.,2804.,2866.
```

TABKA3

```
100 200.,400.,600.,800.,1000.,1500.,2000.,
110 16.,20.,24.,27.,30.,35.,39.
```

PSD

```
$NDM
C     ****************************************************
C     * AUTO-CORRELATION USING FILED DATA                *
C     ****************************************************
      DIMENSION X(1000)
      DIMENSION RX(200),PX(200),SPX(200)
      PRINT ,"INPUT FILE, AUTO-CORR. PLOT FILE, PSD PLOT FILE"
      READ 910, XNAME1,XNAME2,XNAME3
  910 FORMAT(3A6)
      PRINT,"KKK,LAG,DELTAT,FACT"
      READ,KKK,LAG,DELTAT,FACT
      LLAG=LAG+1
      CALL OPENF(1,XNAME1)
      READ(1,920)(X(K),K=1,KKK)
  920 FORMAT(E20.10)
      DO 144 K=1,KKK
  144 X(K)=X(K)*FACT
      PRINT 110
  110 FORMAT(20H RAW TIME SERIES       )
  104 FORMAT(5E12.4)
C     COMPUTE THE MEAN VALUE AND SHIFT THE DATA TO MEAN ZERO
      SUM=0.0
      DO 150 K=1,KKK
  150 SUM=SUM+X(K)
      AKKK=KKK
      XBAR=SUM/AKKK
      PRINT 107,XBAR
  107 FORMAT(6H XBAR=E12.4)
      DO 160 K=1,KKK
  160 X(K)=X(K)-XBAR
      PRINT 103
  103 FORMAT(32H TRANSFORMED INPUT DATA            )
      NPOINT =KKK
      DO 300 I=1,LLAG
      SX=0.0
      M=I-1
      NX=NPOINT -M
      FNX =NX
      DO 305 J=1,NX
      K=M+J
  305 SX=SX+X(J)*X(K)
  300 RX(I)=SX/FNX
      PRINT 400
  400 FORMAT(20H AUTO-CORR. FT.        )
      DO 401 I=1,LLAG,4
      J=I
      J3=I+3
C     PRINT 222,((J,RX(J)),J=I,J3)
  401 CONTINUE
  222 FORMAT(4(I4,E12.4))
```

PSD CONTINUED

```
      PRINT 520,RX(1)
  520 FORMAT(7H RX(1)=E12.4)
      XNAME=XNAME2
C      *******POWER SPECTRAL DENSITY COMPUTATION********
      DELF=1./(DELTAT*AKKK)
      PI=3.1415926536
      FLAG=LAG
      Q=PI/FLAG
      CONST=2.*DELTAT/PI
      RX(1)=0.5*RX(1)
      RX(LLAG)=0.5*RX(LLAG)
      S1=-Q
      DO 1401 IH=1,LLAG
      PX(IH)=0.0
      S1=S1+Q
      S=-S1
      DO 1402 JP=1,LLAG
      S=S+S1
 1402 PX(IH)=PX(IH)+RX(JP)*COS(S)
 1401 PX(IH)=PX(IH)*CONST
     SPX(1)=0.54*PX(1)+0.46*PX(2)
      SPX(LLAG)=0.54*PX(LLAG)+0.46*PX(LLAG-1)
      KK=LLAG-1
      DO 1415 J=2,KK
 1415 SPX(J)=0.54*PX(J)+0.23*(PX(J+1)+PX(J-1))
      WRITE(5,920)(SPX(I),I=1,LLAG)
      PRINT 500
  500 FORMAT(30H POWER SPECTRAL DEN.             )
      XNAME=XNAME3
      IDEV=5
      CALL PLOT4(SPX,LLAG,XNAME,IDEV)
      DO 550 I=1,LLAG
      AI=I
      TEMP13=AI*DELF
      PRINT 554,I,TEMP13,SPX(I)
  550 CONTINUE
  554 FORMAT(I5,E12.4,E16.6)
       PSUM=0.0
      DO 510 I=1,LLAG
  510 PSUM=PSUM+SPX(I)*DELF
      PRINT 511,PSUM
  511 FORMAT(6H PSUM=E12.4)
      END
      SUBROUTINE PLOT4(X,KKK,XNAME,IDEV)
      DIMENSION X(1)
      DIMENSION Y(71),ISYM(71)
      DATA IBLANK/3H    /
      DATA MINUS/3H-   /
      DATA IAI/3HI   /
      DATA IDOT/3H.   /
```

PSD CONTINUED

```
 558 FORMAT(A6)
C     FIND THE MAXIMUN VALUE IN ARRAY X(I)
      TEMP1=0.0
      DO 300 K=1,KKK
      TEMP2=ABS(X(K))
      YAXISL= AMAX1(TEMP1,TEMP2)
C     ::::::::::::::::::::::::::::::::::::::::
 300  TEMP1=YAXISL
      YAXISL=YAXISL+YAXISL/35.
      DELY = YAXISL/35.
      DO 200 IY=1,71
      AIY=IY
      Y(IY)=-YAXISL+DELY*(AIY-1.)
 200  CONTINUE
C      PRINT 215,(Y(IY),IY=1,71)
 215  FORMAT(5E12.4)
 202  FORMAT("*****************************************")
      PRINT 202
      WRITE(IDEV,202)
C      INITIAL PLOT SET
      DO 401 J=1,71
 401  ISYM(J)=MINUS
      ISYM(35)=IAI
      DO 402 K=1,KKK
      DO 403 J=2,71
     IF(Y(J).GT.X(K).AND.Y(J-1).LT.X(K)) GO TO 410
      GO TO 403
 410  ISYM(J-1)=IDOT
 403  CONTINUE
C     PRINT 420,(ISYM(I),I=1,71)
 420  FORMAT(71A1)
C     ++++++++++++++++++++++++++++
      WRITE(IDEV;420)(ISYM(I),I=1,71)
C     +++++++++++++++++++++++++++++
      DO 440 I=1,71
 440  ISYM(I)=IBLANK
      ISYM(35)=IAI
 402  CONTINUE
      PRINT 558,XNAME
      PRINT202
      WRITE(IDEV;202)
C     +++++++++++++++++++++++++++++
      CALL CLOSEF(5,XNAME)
C     +++++++++++++++++++++++++++++
      RETURN
      END
```

TRUCK

```
1$LIB,DIFFEQ
2$LIB,ALGEBR
3$RPC
4$NDM
5$TTY,120
100      COMMON NSTEPS,DELTAT,A,B,MASS,MASS1,MASS2,CPOS1,CPOS2,CNEG1,CNEG2
  110      COMMON C11,C21,INRTIA,FORCW1,FORCW2,SPDEF1,SPDEF2,DSPDF1,DSPDF2
  120      COMMON FORCK1,FORCK2,VAR1,VAR2,VAR3,VAR4,VAR5,VAR6,VAR7,VAR8
130      COMMON ZETA1,ZETA2,DZETA2,ETA1,ETA2,DETA2,NU1,NU2,DNU2
140      COMMON MU1,MU2,DMU2
150      COMMON THRESH(20),GAMMA(20),SEGDEF(20),Y(47)
160      REAL NU1,NU2,MU1,MU2,MASS,MASS1,MASS2,INRTIA
165      IBELL=458752
170      PRINT 1
180    1 FORMAT(18X,33(1H*),/,18X,1H*,31X,1H*,/,
190&       18X,33H*  MURPHY'S M-37 BACK-UP MODEL  *,/,
200&       18X,1H*,31X,1H*,/,18X,33(1H*),///)
210C--------------------DATA INITIALIZATION---------------------
220      A=64.8
230      B=47.2
240      INRTIA=25446.
250      MASS=8.05
260      MASS1=.94
270      MASS2=.8
280      CPOS1=11.8
290      CNEG1=23.8
300      CPOS2=10.2
310      CNEG2=44.
320      THRESH(1)=7.02
330      THRESH(2)=4.5
340      THRESH(3)=2.34
350      THRESH(4)=.81
360      THRESH(5)=.09
370      THRESH(6)=.09
380      THRESH(7)=.81
390      THRESH(8)=2.34
400      THRESH(9)=4.5
410      THRESH(10)=7.02
420      THRESH(11)=7.02
430      THRESH(12)=4.5
440      THRESH(13)=2.34
450      THRESH(14)=.81
460      THRESH(15)=.09
470      THRESH(16)=.09
480      THRESH(17)=.81
490      THRESH(18)=2.34
500      THRESH(19)=4.5
510      THRESH(20)=7.02
520      GAMMA(1)=411.75
530      GAMMA(2)=506.25
```

TRUCK CONTINUED

```
      GAMMA(3)=587.25
      GAMMA(4)=648.
      GAMMA(5)=668.25
      GAMMA(6)=668.25
      GAMMA(7)=648.
      GAMMA(8)=587.25
      GAMMA(9)=506.25
      GAMMA(10)=411.75
      GAMMA(11)=411.25
      GAMMA(12)=506.25
      GAMMA(13)=587.25
      GAMMA(14)=648.
      GAMMA(15)=668.25
      GAMMA(16)=668.25
      GAMMA(17)=648.
      GAMMA(18)=587.25
      GAMMA(19)=506.25
      GAMMA(20)=411.25
      ZETA1=-5.189
      ZETA2=0.
      ETA1=.028
      ETA2=0.
      NU1=-1.08
      NU2=0.
      MU1=-1.245
      MU2=0.
      NSP=0
      POINT1=0.
      POINT2=0.
      SDZ2SQ=0.
      T=0.
      NSTOP=0
      NPL=12
      JJ=1
      DO 10 I=1,47
   10 Y(I)=0.
C----------------------------DATA READ IN----------------------
      PRINT,"************TYPE IN THE DATA FOR THE FOLLOWING"
      PRINT,
      PRINT,"EXTERNAL INPUT DATA"
      PRINT,
      PRINT,"NAME OF PROFILE INPUT FILE"
      READ 2,FINAME
    2 FORMAT(A6)
      PRINT,"DATA VARIABLES"
      PRINT,
      PRINT,"TRUCK VELOCITY IN INCHES PER SECOND   [REAL]"
      READ,VEL
      PRINT,"PROGRAM VARIABLES"
      PRINT,
```

TRUCK CONTINUED

```
     PRINT,"NUMBER OF SPACES BETWEEN PROFILE POINTS  [INTEGER]"
     READ,NSPACE
     PRINT,"NUMBER OF STEPS IN RKG   [INTEGER]"
     READ,NSTEPS
     PRINT,"EXTERNAL OUTPUT DATA"
     PRINT,
     PRINT,"NAME OF OUTPUT DATA FILE"
     READ 2,FNAME2
     PRINT,
     PRINT,"********END OF DATA INPUT*********"
     PRINT 3
     XMPH=VEL/17.6
   3 FORMAT(/////,"T",10X,"Y(1)",7X,"AZETA2",5X,
&     "ARMS",/)
     WRITE(2;6)
   6 FORMAT(//,37X,45(1H*),/,37X,1H*,43X,1H*,/,37X,
&        45H*  MURPHY'S M-37 TRUCK PROGRAM OUTPUT FILE  *,/,
&        37X,1H*,43X,1H*,/,37X,45(1H*),///)
     WRITE(2;11)XMPH,NSTEPS
     WRITE(2;9)
  11 FORMAT(/,9HVELOCITY=,F6.2,15H MILES PER HOUR,50X,
&       35HNUMBER OF STEPS IN RKG INTEGRATION=,I4)
   9 FORMAT(//,13X,9(1H-),12HDISPLACEMENT,8(1H-),3X,10(1H-),
&        8HVELOCITY,11(1H-),3X,9(1H-),12HACCELERATION,
&        8(1H-),2X,7HC-G RMS,/,X,4HTIME,2X,4HY(1),X,
&        3(2X,3HC-G,4X,5HPITCH,3X,5HFR-AX,3X,5HRE-AX,2X),
&        X,5HACCEL,//)
     DELTAT=3.07/VEL
     CALL OPENF(1,FINAME)
   4 FORMAT(E20.10)
  50 POINT1=POINT2
     IF(JJ-2)55,40,55
  55 READ(1,4)POINT2
     CALL EOFTST(1,JJ)
     GO TO(60,40)JJ
  40 POINT2=POINT1
     NSTOP=NSTOP+1
     NSPACE=1
  60 PSTEP=(POINT2-POINT1)/NSPACE
     NSP=0
 100 I=46
  70 Y(I+1)=Y(I)
     I=I-1
     IF(I)70,75,70
  75 Y(1)=POINT1
     POINT1=POINT1+PSTEP
     NSP=NSP+1
     CALL DIFFEQ
     SDZ2SQ=SDZ2SQ+DZETA2*DZETA2
     RMSDZ2=SQRT((SDZ2SQ*DELTAT)/T)
```

TRUCK CONTINUED

```
    5 FORMAT(6(X,G10.4))
      NPL=NPL+1
      AZETA2=DZETA2/386.
      ANU2=DNU2/386.
      AMU2=DMU2/386.
      RMSAZ2=RMSDZ2/386.
      WRITE(2;7)T,POINT1,ZETA1,ETA1,MU1,NU1,ZETA2,ETA2,MU2,NU2,
&       AZETA2,DETA2,AMU2,ANU2,RMSAZ2
    7 FORMAT(F6.1,F5.1,13F8.3)
      IF(NPL-54)120,110,120
  110 WRITE(2;9)
      PRINT 5,T,Y(1),AZETA2,RMSAZ2
      NPL=0
  120 IF(NSTOP-47)80,90,80
   80 T=T+DELTAT
      IF(NSP-NSPACE)100,50,100
   90 CALL CLOSEF(2,FNAME2,2)
     PRINT 12,(IBELL,I=1,40)
   12 FORMAT(40A1)
      PRINT 8,FNAME2
    8 FORMAT(///,"DETAILED OUTPUT IS IN FILE",X,A6,/)
      CALL EXIT
      END
```

DIFFEQ

```
      SUBROUTINE DIFFEQ
      REAL NU1,NU2,MU1,MU2,MASS,MASS1,MASS2,INRTIA
      H=DELTAT/NSTEPS
      RH=1./H
      INDEX=0
  100 INDEX=INDEX+1
      VAR1=ZETA1
      VAR2=ZETA2
      VAR3=ETA1
      VAR4=ETA2
      VAR5=NU1
      VAR6=NU2
      VAR7=MU1
      VAR8=MU2
      PZETA2=ZETA2
      PETA2=ETA2
      PNU2=NU2
      PMU2=MU2
      CALL ALGEBR
      FK11=H*VAR2
      FK12=H*(1/MASS)*(FORCK1+C11*DSPDF1+FORCK2+C21*DSPDF2-MASS*386.0)
      FK13=H*VAR4
      FK14=H*(1/INRTIA)*(A*FORCK1+A*C11*DSPDF1-B*FORCK2-B*C21*DSPDF2)
      FK15=H*VAR6
      FK16=H*(1/MASS1)*(-FORCK1-C11*DSPDF1+FORCW1-MASS1*386.0)
      FK17=H*VAR8
      FK18=H*(1/MASS2)*(-FORCK2-C21*DSPDF2+FORCW2-MASS2*386.0)
      VAR1=ZETA1+FK11*.5
      VAR2=ZETA2+FK12*.5
      VAR3=ETA1+FK13*.5
      VAR4=ETA2+FK14*.5
      VAR5=NU1+FK15*.5
      VAR6=NU2+FK16*.5
      VAR7=MU1+FK17*.5
      VAR8=MU2+FK18*.5
      CALL ALGEBR
      FK21=H*VAR2
      FK22=H*(1/MASS)*(FORCK1+C11*DSPDF1+FORCK2+C21*DSPDF2-MASS*386.0)
      FK23=H*VAR4
      FK24=H*(1/INRTIA)*(A*FORCK1+A*C11*DSPDF1-B*FORCK2-B*C21*DSPDF2)
      FK25=H*VAR6
      FK26=H*(1/MASS1)*(-FORCK1-C11*DSPDF1+FORCW1-MASS1*386.0)
      FK27=H*VAR8
      FK28=H*(1/MASS2)*(-FORCK2-C21*DSPDF2+FORCW2-MASS2*386.0)
      VAR1=ZETA1+.29289322*FK21+.20710678*FK11
      VAR2=ZETA2+.29289322*FK22+.20710678*FK12
      VAR3=ETA1+.29289322*FK23+.20710678*FK13
      VAR4=ETA2+.29289322*FK24+.20710678*FK14
      VAR5=NU1+.29289322*FK25+.20710678*FK15
      VAR6=NU2+.29289322*FK26+.20710678*FK16
```

DIFFEQ CONTINUED

```
      VAR7=MU1+.29289322*FK27+.20710678*FK17
      VAR8=MU2+.29289322*FK28+.20710678*FK18
      CALL ALGEBR
      FK31=H*VAR2
      FK32=H*(1/MASS)*(FORCK1+C11*DSPDF1+FORCK2+C21*DSPDF2-MASS*386.0)
        FK33=H*VAR4
      FK34=H*(1/INRTIA)*(A*FORCK1+A*C11*DSPDF1-B*FORCK2-B*C21*DSPDF2)
        FK35=H*VAR6
      FK36=H*(1/MASS1)*(-FORCK1-C11*DSPDF1+FORCW1-MASS1*386.0)
      FK37=H*VAR8
      FK38=H*(1/MASS2)*(-FORCK2-C21*DSPDF2+FORCW2-MASS2*386.0)
      VAR1=ZETA1-.70710678*FK21+1.70710678*FK31
      VAR2=ZETA2-.70710678*FK22+1.70710678*FK32
      VAR3=ETA1-.70710678*FK23+1.70710678*FK33
      VAR4=ETA2-.70710678*FK24+1.70710678*FK34
      VAR5=NU1-.70710678*FK25+1.70710678*FK35
      VAR6=NU2-.70710678*FK26+1.70710678*FK36
      VAR7=MU1-.70710678*FK27+1.70710678*FK37
      VAR8=MU2-.70710678*FK28+1.70710678*FK38
      CALL ALGEBR
      FK41=H*VAR2
      FK42=H*(1/MASS)*(FORCK1+C11*DSPDF1+FORCK2+C21*DSPDF2-MASS*386.0)
        FK43=H*VAR4
      FK44=H*(1/INRTIA)*(A*FORCK1+A*C11*DSPDF1-B*FORCK2-B*C21*DSPDF2)
        FK45=H*VAR6
      FK46=H*(1/MASS1)*(-FORCK1-C11*DSPDF1+FORCW1-MASS1*386.0)
      FK47=H*VAR8
      FK48=H*(1/MASS2)*(-FORCK2-C21*DSPDF2+FORCW2-MASS2*386.0)
      ZETA1=ZETA1+FK11*.166667+.09763107*FK21+.56903559*FK31+FK41*.1666
        ZETA2=ZETA2+FK12*.166667+.09763107*FK22+.56903559*FK32+FK42*.16
        ETA1=ETA1+FK13*.166667+.09763107*FK23+.56903559*FK33+FK43*.1666
        ETA2=ETA2+FK14*.166667+.09763107*FK24+.56903559*FK34+FK44*.1666
        NU1=NU1+FK15*.166667+.09763107*FK25+.56903559*FK35+FK45*.166667
        NU2=NU2+FK16*.166667+.09763107*FK26+.56903559*FK36+FK46*.166667
        MU1=MU1+FK17*.166667+.09763107*FK27+.56903559*FK37+FK47*.166667
        MU2=MU2+FK18*.166667+.09763107*FK28+.56903559*FK38+FK48*.166667
        DZETA2=(ZETA2-PZETA2)*RH
      DETA2=(ETA2-PETA2)*RH
      DNU2=(NU2-PNU2)*RH
      DMU2=(MU2-PMU2)*RH
       IF(INDEX-NSTEPS)100,200,200
  200 RETURN
       END
```

ALGEBR

```
      SUBROUTINE ALGEBR
      REAL NU1,NU2,MU1,MU2,MASS,MASS1,MASS2,INRTIA
      DO 100 I=1,10
      SEGDEF(I)=Y(I)-VAR5-THRESH(I)
      IF(SEGDEF(I))50,100,100
   50 SEGDEF(I)=0.
  100 CONTINUE
      DO 200 I=11,20
      SEGDEF(I)=Y(I+26)-VAR7-THRESH(I)
      IF(SEGDEF(I))150,200,200
  150 SEGDEF(I)=0.
  200 CONTINUE
      FORCW1=0.
      FORCW2=0.
      DO 300 I=1,10
      FORCW1=FORCW1+GAMMA(I)*SEGDEF(I)
  300 CONTINUE
      DO 400 I=11,20
      FORCW2=FORCW2+GAMMA(I)*SEGDEF(I)
  400 CONTINUE
      SPDEF1=VAR5-VAR1-A*SIN(VAR3)
      SPDEF2=VAR7-VAR1+B*SIN(VAR3)
      DSPDF1=VAR6-VAR2-A*VAR4*COS(VAR3)
      DSPDF2=VAR8-VAR2+B*VAR4*COS(VAR3)
      FORCK1=19.54*(SPDEF1**3)-192.42*SPDEF1*SPDEF1+913.55*SPDEF1
      FORCK2=1.39*(SPDEF2**3)-1.24*SPDEF2*SPDEF2+307.72*SPDEF2
      IF(DSPDF1)600,500,500
  500 C11=CPOS1
      GO TO 700
  600 C11=CNEG1
  700 IF(DSPDF2)800,900,900
  900 C21=CPOS2
      RETURN
  800 C21=CNEG2
      RETURN
      END
```

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13. ABSTRACT

The specific objective of this study was to define and formulate the procedures for implementing a practical, compact description of terrain-vehicle-speed (TVS) systems and thus establish a methodology for relating measures of ride to measures of terrain roughness and cross-country speed. This methodology should enable the determination of reliable estimates of the average speed a given vehicle can attain under given vibrational constraints as well as the probabilities of exceeding certain specified levels. This study was entirely a computerized effort. Digital computer simulations were conducted in which a vehicle was run at several selected speeds over terrain profiles with various levels of roughness. The terrain profiles were generated by computer programs that constructed and shaped sequences of random normal numbers to provide representative profiles with a desired power spectrum, variance, mean value, standard deviation, and RMS. These programs also performed the necessary statistical checks for normality, stationarity, and randomness. A comprehensive, nonlinear mathematical model of an M37 truck served as the vehicle. This model allowed for the nonlinearities inherent in large rotational motions, the suspension elements, bump stop, loss of ground contact, and the tire compliance, which was represented by a cluster of radially projecting springs. The results are presented in the form of three basic graphs comparing input and output statistics and distribution functions. The input statistics consisted of terrain roughness as measured by RMS elevation. The output is represented as the RMS vertical acceleration of the vehicle's center of gravity. A family of curves was developed relating vehicle response to terrain roughness for each speed of vehicle traversal. Cross plots of these established relations provided a more useful relation between vehicle response and speed for various degrees of terrain roughness. The use of such relations to catalog vehicles in terms of their ride dynamics capabilities is suggested. The TVS scheme is readily adaptable to computer models for evaluating and rating the cross-country mobility characteristics of vehicles traversing large, nonhomogeneous terrain areas.

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